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A MODIFIED INTERPOLATION APPROACH FOR TOPOLOGY OPTIMIZATION**

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ABSTRACT In view of the fact that the follow-up search for an optimal topology is affected by deleting a large number of high-relative-density elements. When the typical density interpolation approach, namely, solid isotropic microstructures with penalization (SIMP), is employed in the continuum structural topology optimization, a new density interpolation approach based on the logistic function is proposed in this paper. This method can weaken low-relative-density elements while enhancing high-relative-density elements by polarization, and then rationally realize polarization of the intermediate density elements. It can reduce the number of gray-scale elements as much as possible to get the optimal topology with distinct boundaries in conjunction with the sensitivity filtering method based on particle swarm optimization (PSO). Several typical numerical examples are given to demonstrate this method.

KEY WORDS topology optimization density interpolation approach Logistic function, gray-scale element

I. INTRODUCTION

Topology optimization has become one of the fastest growing research fields by virtue of its broad applications in areas as diverse as structures (e.g., Li[1] and Hu[2]), mechanisms and actuators (e.g., Du[3] and Luo[4]), and materials (e.g., Huang[5] and Wang[6]). One of the main targets of structural topology optimization is to determine the best distribution of material within a given domain[7] Structural topology optimization aims at finding the optimum distribution of material within a specified domain. Hence, it determines which parts of the domain should contain material (i.e., structure) and which should remain void. Many researchers have studied this area, and many approaches have been used in an attempt to solve this problem. Xie[8] proposed the Evolutionary Structural Optimization (ESO) method in 1993, which received wide consideration, although this method has been controversial for lack of rigorous mathematical foundation[9]. As Bendsøe[10] clarified the physical relevance of different material interpolation schemes in 1999, the solid isotropic material with penalization (SIMP) method attracted much attention and found engineering application[11]. Recently, the level-set based method (LSM) has aroused great interest in the area of structural optimization (e.g., Allaire[12] and Luo[13]). In addition, the Independent Continuous Mapping (ICM) method[14,15] proposed by Sui, the pointwise

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density-based interpolation (PDI) method (e.g., Wang[16] and Luo[17]), and various uncertainty methods (e.g., Luo[18] and Luo[19]) have also been developed for topology optimization.

However, despite its rapid development, structural topology optimization still needs further research. A topology optimization problem in a continuum is originally a combinatorial optimization problem with 0 and 1 discrete design variables, to which few gradient-based optimization algorithms can be directly applied. Rather, it is apt to bring about combination explosion if solved as a 0-0 programming problem. To overcome this limitation, the most commonly used approach is to replace the integer 0-1 variables with continuous variables whose values range from 0 to 1, and then introduce some form of penalty to steer the solution to discrete 0-1 values. This replace-and penalize approach, often called the interpolation scheme, depends on the type of a material model whose nature characterizes the different approaches to topology optimization. Therefore, topology optimization is a two-step process: of creating a material model (formulation) followed by finding methods to resolve it (solution). As an extension of the homogenization method, the solid isotropic material penalization (SIMP) method has become popular since it considers only a density of the given materials to avoid the concept of homogenization theory for composites. This method can effectively solve the difficulty in obtaining the optimal solution to the problem with discrete design variables by establishing the corresponding relation between the relative density and Young's Modulus. And an exponential 'power-law' scheme[10] is usually involved to penalize the intermediate densities.

However, in the SIMP interpolation scheme, to suppress the intermediate density elements as much as possible in the final optimal topology, the penalty factor should be as large as possible theoretically. A small penalty factor will lead to a large number of intermediate density elements as the punishment is insufficient in the process of solving. And it is common to generate designs involving gray-scale elements, as a portion of material with intermediate densities will appear to surround structural boundaries. This makes it hard to accurately interpret the final topological design because of the fuzzy boundaries, as under- or over-evaluated structural boundaries are undesirable. When the penalty parameter is large enough, the optimal topology will be closer to 0-1 discrete design variables. But many high-relative-density elements will be penalized to 0 during the process of solution. This phenomenon is detrimental to the topological optimization of continuum structure. First of all, at the beginning of the model's solution, most of the elements' relative densities are between 0 and 1.0, too many elements are deleted because they are compelled to 0 during the process of iteration. The direct result is the follow-up search for the optimal topology is affected,to the effect that the final result is not optimal. Moreover, it is irrational to delete many high-relative-densities elements (for example, greater than 0.5 and less than 1.0) in the process of topology optimization. And if the penalty factor is too large, it will lead to checkerboard pattern numerical instability.

In order to make up for the deficiencies of the SIMP method and obtain rational results, there have been a lot of research endeavors with a view to solving this problem. For instance, Fuchs[20] obtained relative clear 0-1 topological structure by introducing the sum of the reciprocal variables (SRV) constraints to the SIMP approach and combine with the method of moving asymptotes (MMA) arithmetic; Wang[?]considered the density gradient information and proposed a bilateral filtering method to get black and white topology optimization results by weakening the average filtering effect on the smaller density gradient of the model. Groenwold[22] put forward a modified heuristic optimality criterion method combined with the SIMP scheme to suppress the gray-scale elements, and get a relatively good topology structure. Kang[23] proposed a nodal-density based interpolation scheme in order to achieve a better black-white boundary, and Luo[17] proposed a pointwise meshless interpolation model to avoid intermediate densities around the design boundary of the structure.

Considering these limitations in SIMP method, this paper introduces a modified material interpolation method for topology optimization based on the logistic regression analysis function, which is able to establish a more rational relation between the material densities and Young's modulus. It can weaken the low-relative-density elements while enhancing the high-relative-density elements when used to lead the topology optimization, and then rationally realize the polarization of the intermediate density elements. When this method is combined with the sensitivity filtering method based on the particle swarm optimization (PSO)[24,25], one can get better optimal topologies with distinct boundaries as the intermediate-density elements were extremely reduced during the process of solving.

The following sections are arranged as: §II describes the modified density interpolation method; §III tells the topology optimization model and the solution method; and §IV presents some numerical examples to show the efficiency and accuracy of the modified density interpolation method in solving topology optimization problems and makes some comparison; ultimately, §V presents some discussion and conclusions.

II. MODIFIED DENSITY INTERPOLATION METHOD

In SIMP, a density-stiffness interpolation scheme is used to represent the nonlinear dependency between elemental densities and material properties. To recover the original 0 and 1 discrete material distribution, a power-law scheme[10] is usually adopted to penalize the intermediate densities to push the intermediate densities towards its binary bounds (0/1). In most engineering applications, the SIMP method can be generally written as

$$E_e(x_e) = x_e^p E_0 \tag{1}$$

where E_e and E_0 denote the actual and initial Young's modulus, respectively, p is the penalty factor. The density penalty function of this model is

$$f(x_i) = x_i^p \tag{2}$$

where x_i is the elements' relative density.

Figure 1 shows the different shapes of penalty effect when the penalty factor p is endowed with different values. From the figure, one can see that during the punishment in the SIMP approach, most of the intermediate density elements tend to 0. For example, p is 3.0, the amounts of the elements' densities are between 0.4 and 0.9, when the transformation of Eq.(1) is employed, the corresponding Young's modulus of relative materials is confined to between 0.08 and 0.72. After many times of iteration, nearly all these elements will quickly tend to 0. So, only a few high-relative-density elements (the relative density close to 1.0) will tend to 1.0 by polarization. This phenomenon will affect the follow-up search for the optimal topology as the high-relative-density elements deleted are not able to contribute to the whole process.

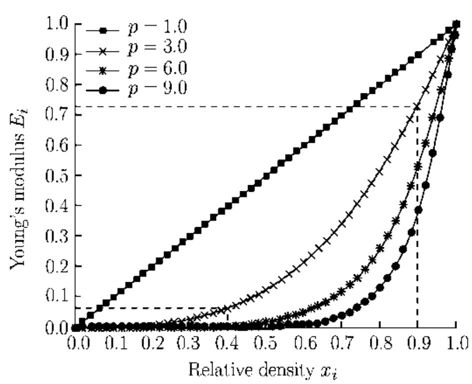


Fig. 1. SIMP model with different penalty factors.

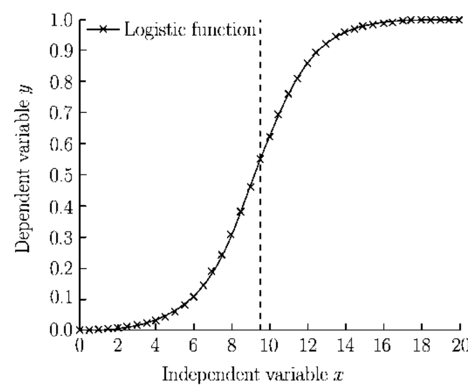


Fig. 2. Logistic function with single independent variable.

Considering the drawback of the SIMP approach, this paper will propose a modified density interpolation method based on the Logistic regression analysis function. Logistic function is a kind of regression analysis model for predicting the probability of occurrence of dependent variables through multi-independent variables. And it is evolved into Eq.(3) if it possesses a single variable. Figure 2 shows the graph of Eq.(3) when the parameters a and b are -6 and 0.65, respectively.

$$y = \frac{\mathrm{e}^{a+bx}}{1+\mathrm{e}^{a+bx}} \qquad (a, b \in R) \tag{3}$$

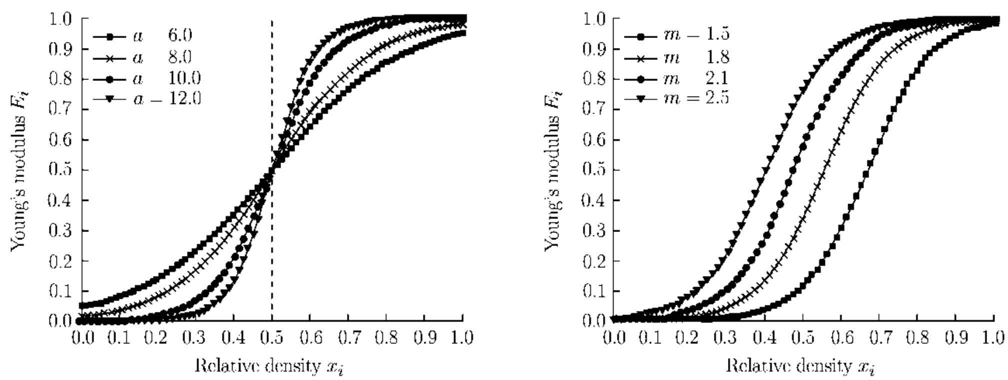
Figure 2 indicates that when the independent variable moving toward the two sides from the center line, respectively , the dependent variable will tend to 0 or 1.0, respectively, namely, this function can realize the feature that the variable will be weakened and tends to 0 if its value is smaller than the center line's value, otherwise, will be enhanced and tends to 1.0. Based on this merit of polarization, this paper proposed a modified density interpolation method. This methodology can weaken the low-relative-density elements while enhancing the high-relative-density elements by polarization, and then rationally realize the polarization of the intermediate density elements to obtain a better design in topology optimization. It can be written as

$$f(x_i) = \frac{\mathrm{e}^{-a/m+ax_i}}{1+\mathrm{e}^{-a/m+ax_i}} \tag{4}$$

where $f(x_i)$ is element's relative density after updating; a is the penalty factor used to control the punishment speed of the elements that belong to the middle part; m is a density boundary factor. When the element's relative density is less than 1/m, it will be weakened and tend to 0 after being transformed by Eq.(4), otherwise, it will be enhanced and tend to 1.0.

Substitute Eq.(4) into Eq.(1), then obtain the density interpolation model's expression between Young's modulus and the relative density

$$E_i = \frac{\mathrm{e}^{-a/m+ax_i}}{1+\mathrm{e}^{-a/m+ax_i}} E_0 \tag{5}$$



(a) $m = 2.0$, $a = 6.0, 8.0, 10.0, 12.0$ (b) $a = 12.0$, $m = 1.5, 1.8, 2.1, 2.5$

Fig. 3. The proposed density interpolation model.

Figure 3 shows the interpolation curves with different parameters a and m. Figure 3(a) indicates the relation of the Young's Modulus, the element's relative density and the penalty factor when the density boundary factor is 2.0. We can find that the curves become steeper and steeper with the penalty factor increasing other things being equal. It can be seen from Fig.3(b) that when the density boundary factor is changing, the punishment curve will regard the different relative density as the demarcation point and make the intermediate densities materials approach 0 or 1. When m is increasing, the demarcation point will move toward the direction of 0, otherwise, toward the direction of 1.0. To different engineering applications, one can get better optimal topologies by setting different relative density demarcation points.

As indicated in Fig.1, the SIMP scheme will delete too many high-relative-density elements, and it will affect the follow-up search of the optimal topology, what's more, since the number of the intermediate density is relatively large, it is inevitably to produce many gray-scale elements in the final optimal topology after the punishment. While Fig.3(a) shows that the proposed method can polarize the majority of elements to 0 or 1.0 more rapidly at the same probability. So the number of elements with the intermediate density in the final topology will be sharply reduced, so as to get a topological design with distinct boundaries.

To illustrate the superiority of the proposed method, both the original SIMP scheme and the proposed method's sensitivity will be analyzed further. From Eq.(1) and Eq.(5), the sensitivities of the two interpolation models can be respectively written as

$$\frac{\partial E_i}{\partial x_i} = p x_i^{p-1} E_0 \tag{6}$$

$$\frac{\partial E_i}{\partial x_i} = a \frac{\mathrm{e}^{-a/m+ax_i}}{(1+\mathrm{e}^{-a/m+ax_i})^2} E_0 \tag{7}$$

Figures 4 and 5 are the corresponding graphs of Eqs.(6) and (7), respectively.

Figure 4 shows that with an increase of the penalty factor, the elements whose sensitivity is 0 are also gradually increase. For example, when the penalty factor is 3.0, 6.0 and 9.0, the elements' relative density whose sensitivities are 0 are between 0 and 0.2, 0 and 0.45, 0 and 0.52, respectively.

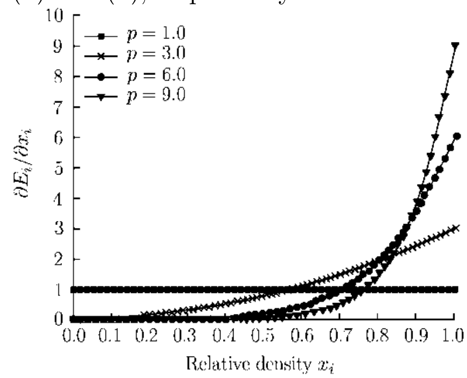


Fig. 4 Sensitively of SIMP model (p = 1.0, 3.0, 6.0, 9.0).

In the interpolation model based on continuous variable, many mature gradient-based optimization algorithms are directly applied during the process of solving. When there are too many elements whose Young's modulus' derivatives are 0, it means that these elements can't continue to guide the subsequent optimization process. So, as shown in Fig.4, with an increase of the penalty factor, the number of deleted elements also gradually increase, and most of them with relative densities between 0.5 and 1.0 will tend to 0 by polarization with the iteration, which will inevitably affect the follow-up search for the optimal topology and the calculation result can hardly guarantee accuracy. While Fig.5 shows that in the proposed approach, the Young's modulus' sensitivity curves move along the horizontal direction, and most important, nearly none of the elements' sensitivity information is 0 no matter how the penalty factor changes. This demonstrates that when the modified methodology is employed to guide topology optimization, all elements in the design domain contribute to the whole process of topology optimization.

III. TOPOLOGY OPTIMIZATION MODEL AND THE SOLUTION METHOD

3.1. Optimization Formulation with SIMP

In the SIMP method, the design domain is discretized into finite element (FE) meshes defined by N_x, which is the set of elements in the x-axis ($N_x = \{1, 2, \ldots, |N_x|\}$) and N_y, which is the set of elements in the y-axis ($N_y = \{1, 2, \ldots, |N_y|\}$). The relative density of every element in the mesh (x_i, $i \in N_x \times N_y$) is considered a design variable ($0 \leq x_i \leq 1$). It is assumed by SIMP method that the stiffness matrix of each element depends on the relative density raised to some penalization power, p.

In this paper, based on Eq.(5) the optimization problem of the minimum compliance can be written as

$$\begin{aligned} &\text{Find}: \quad x = (x_1,\, x_2,\, x_3,\, \ldots,\, x_n)^{\mathrm{T}} \\ &\text{Min}: \quad c(x) = \boldsymbol{U}^{\mathrm{T}}\boldsymbol{K}\boldsymbol{U} = \sum_{i=1}^{N} \frac{\mathrm{e}^{-a/m+ax_i}}{1+\mathrm{e}^{-a/m+ax_i}} E_0 u_i^{\mathrm{T}} k_i u_i \\ &\text{Subject to}: \ V(x)/V_0 \leq f \\ &\qquad\qquad \boldsymbol{K}\boldsymbol{U} = \boldsymbol{F} \\ &\qquad\qquad 0 \leq x_{\min} \leq x_{\max} \leq 1 \end{aligned} \tag{8}$$

where$x = (x_1, x_2,\, \ldots,\, x_N)^{\mathrm{T}}$ is an N-dimensional vector of the design variables; $c(x)$ is the compliance given a topology defined by the density vector x of decision variables x_i; $\boldsymbol{U}$ and $\boldsymbol{F}$ are the global displacement and force vectors, respectively; $\boldsymbol{K}$ is the global stiffness matrix; $N(N = N_x \times N_y)$ is the number of elements used to discretize the design domain, namely, the number of the design variables; u_i is the element displacement vector, k_i is the element stiffness matrix; f is the prescribed volume fraction; $x_{\min}$ and $x_{\max}$ are the lower and upper bounds of the relative densities (non-zero to avoid singularity, typically the value); $V(x)$ and V are the material volume and the design domain volume, respectively.

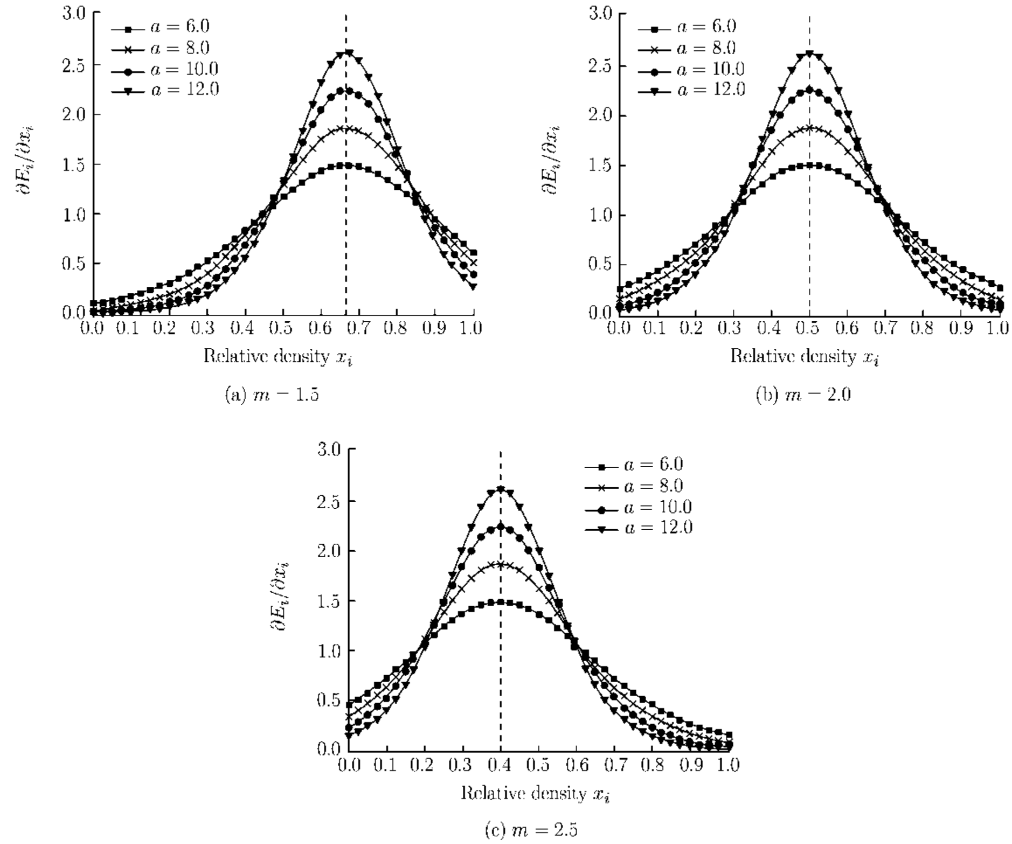


Fig. 5. Sensitively of the proposed density interpolation model ($m = 1.5, 2.0, 2.5$).

3.2. Optimality Criterion Method

The Optimality Criterion (OC)[26] method realizes the optimization solution through establishing the optimality criterion and the iteration formula. And it has many advantages if used: fast convergence, the complexity level has no association with the structural reanalysis and the number of the variables. In this paper, the method of updating element density is based on the OC method, which can be written as

$$x_i^{n+1} = \begin{cases} \max(x_{\min}, x_i^n - t), & x_i^n B^\eta < \max(x_{\min}, x_i^n - t) \\ x_i^n B^\eta, & \max(x_{\min}, x_i^n - t) < x_i^n B^\eta < \min(1, x_i^n + t) \\ \min(1, x_i^n + t), & x_i^n B^\eta > \min(1, x_i^n + t) \end{cases} \tag{9}$$

where t is a positive move-limit, η is a numerical damping coefficient (typically the value is 1/2), n is the iterations and B_e is defined as $B = -\dfrac{1}{\lambda}\dfrac{\partial c/\partial x_i}{\partial V/\partial x_i}$, λ is a Lagrangian multiplier that can be found by a bi-sectioning algorithm.

3.3. Sensitivity Analysis

Based on filtering techniques from image processing, the sensitivity filtering scheme[27] has been widely used to avoid numerical instabilities such as the checkerboards and mesh-dependency in the topology optimization of continua, which modified the design sensitivities during iterations as follows:

$$\widehat{\frac{\partial c}{\partial x_e}} = \frac{1}{x_e \sum_{f=1}^{N} \hat{H}_f} \sum_{f=1}^{N} \hat{H}_f x_f \frac{\partial c}{\partial x_f} \tag{10}$$

The convolution operator (weight factor) $\hat{H}_f$ is written as

$$\begin{aligned} &\hat{H}_f = r_{\min} - \mathrm{dist}(e, f) \\ &\{f \in N \,|\mathrm{dist}(e, f) \leq r_{\min}\}, \quad e = 1, ..., N \end{aligned} \tag{11}$$

where the operator dist$(e,\ f)$ is defined as the distance between the center of elements e and f, $r_{\min}$ is the filter radius.

This method makes the design sensitivity of a specific element dependent on a weighted average around the element's neighbors located within the range of the radius dist(e, f). Because of the weighted average operation, a specific element's density will be repeatedly evaluated many times. The direct side-effect is it will lead to gray-scale elements with intermediate densities, although is it can avoid the checkerboards efficiently. In particular, with ane increase of the radius dist$(e,\ f)$, the gray-scale becomes even more serious.

The Particle Swarm Optimization (PSO)[24] algorithm is a swarm intelligence algorithm originally proposed by Kennedy and Eberhart in 1995 and inspired by swarm behaviors such as birds flocking and fish schooling. Each particle flies in the search space and adjusts its flying trajectory according to its personal best experience and its neighborhood's best experience. Owing to its simple concept and high efficiency, PSO has become a widely adopted optimization technique and has been successfully applied to many real-world problems.

According to the flying rule of the particle in the standard PSO algorithm, a modified updating method is proposed to modify the sensitivities information of discrete elements. Compared to the sensitivity filtering technique, this scheme modifies the element design sensitivity depending on its own sensitivity, its neighborhood's maximal sensitivity and minimal sensitivity, which is expressed as

$$\frac{\widehat{\partial c}}{\partial x_e} = \omega \frac{\partial c}{\partial x_e} + c_1 \max_{i \in \hat{N}} \frac{\partial c}{\partial x_i} + c_2 \min_{j \in \hat{N}} \frac{\partial c}{\partial x_j} \tag{12}$$

where coefficients c_1 and c_2 are positive learning factors (also called acceleration parameters), ω is the inertia weight and $\hat{N}$ is the number of neighboring elements. In this research, $\hat{N} = 8$ which is shown in Fig.6 is used.

In Eq.(12), the inertia weight ω is employed to control the impact of previous element sensitivity on the filtered sensitivity; the learning factors c_1 and c_2, however, are used to control the impact of maximal and minimal sensitivity around eight neighboring elements on filtered sensitivity. According to plenty of numerical experimental results, a larger inertia weight ω facilitates local exploration and may produce checkerboards while a smaller inertia weight ω tends to facilitate global exploration to fine-tune the current filtered sensitivities and slow down the convergence rate. Larger learning factors facilitate large fluctuation of element sensitivities while smaller learning factors tend to facilitate global exploration and produce checkerboards. Suitable selection of the inertia weight ω, the learning factors c_1 and c_2 can provide a balance between global and local exploration abilities and thus require relatively lesser iterations on convergence to find the optimum. The inertia weight $\omega = 0.2$ and the learning factors $c_1 = c_2 = 0.4$ are used for better optimization effect in this research.

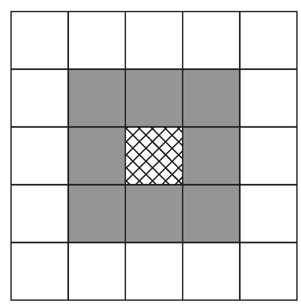


Fig. 6 Model of eight boundary elements.

IV. NUMERICAL EXAMPLES

Numerical examples in two-dimensions are presented in this section to demonstrate the availability, the efficiency and the stability of the process control of the proposed method in this paper. In each example, the design domain is discretized into four-node quadrilateral finite elements of low order. The convergence criterion for these examples is satisfied when the prescribed change of structural compliance is less than 0.1%. And the two examples are both about problems with single load case and single constraint condition, the load F is 1, material's Young's modulus E is 1, and the Poisson's ratio μ is

0.3 (relative value and dimensionless).

4.1. The Optimization of the Cantilever

As indicated in Fig.7, the design domain of the problem is a 4.8 m×3.0 m rectangle with a thickness of 0.01 m, discretized into 48×30 elements during structural analysis. The load F is located midway on the right boundary line, and the whole left boundary line is constrained.

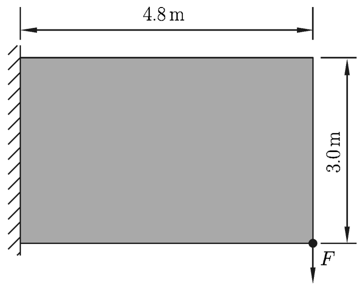


Fig. 7. Design domain with working conditions of the cantilever.

Table 1. **Numerical example one's optimal topologies of different volume fractions**

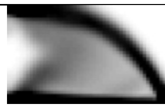
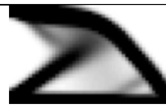
Volume fraction f	0.4	0.5	0.6
SIMP method			
The proposed method			

Table 2. **Numerical example one's comparison of the optimization results**

	Volume fraction f	Compliance C	Gray-scale elements ($0.2 \leq x_i \leq 0.8$)
0.4	SIMP method	60.0757	198
	The proposed method	58.9321	32
0.5	SIMP method	48.9457	220
	The proposed method	47.9379	32
0.6	SIMP method	42.1413	206
	The proposed method	41.5550	28

Tables 1, 2 and 3 are the optimal topologies and corresponding results with different methods. From Table 1, it can be seen that the modified interpolation method can accurately produce optimal topologies, and the major merit is the boundary of final topological designs is distinct because of the intermediate density elements are greatly polarized approach to 0 or 1.0. As mentioned in §II, in the updating elemental densities, Eq.(5) can reasonably make the design variables close to 0 or 1.0, instead of deleting too many elements of high-relative-density between 0.5 and 1.0, which affect the follow-up search of the optimal topology. Furthermore, since the modified sensitivity approach is adopted in this paper to realize the sensitivity filtering, the sensitivity information is rationally updated. These might be the reasons why the compliances are lower and the gray-scale elements are lesser than the conventional SIMP method's that Table 2 indicates. As shown in Table 2, the conventional SIMP method

Table 3. **Snapshots of the topology obtained at different iteration steps t with $f = 0.5$**

Steps	$t = 5$	$t = 10$	$t = 15$	$t = 20$	Optimal topologies
SIMP method					
The proposed method					

leads to 198, 220 and 206 gray-scale elements with intermediate material densities (relative density between 0.2 and 0.8) in total, respectively when combined with the conventional filtering method, while the modified interpolation method yields 32, 32 and 28 in this given problem in conjunction with the modified sensitivity analysis method as mentioned in §3.3, respectively. Table 3 is the snapshots of the topological designs obtained at different iteration steps t when the volume fraction f is 0.5, showing that the proposed approach can generate topology design using the stable control process.

4.2. The Optimization of the Half MBB-beam

The second example is shown in Fig.8, which is further used to demonstrate the effectiveness of the proposed material interpolation method. In the numerical implementation, only half the 'MBB-beam' is used to take advantage of structural symmetry. In the symmetric design domain, the load is applied vertically in the upper left corner and the lower right corner is simply supported, and the left edge is regarded as the symmetric boundary condition. The design domain is a 6.0 m×2.0 m rectangle area with a thickness of 0.01 m, discretized by 60×20 quadrilateral finite elements for structural analysis, three volume fractions ($f = 0.4$, 0.5, 0.6) are employed as the volume constraints in the process of each solution , respectively.

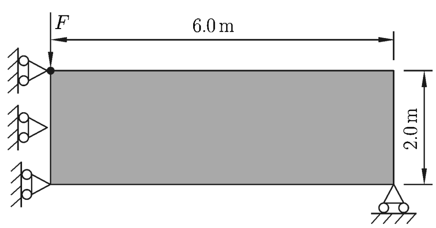
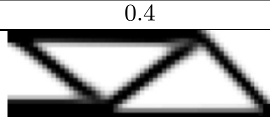
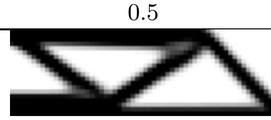
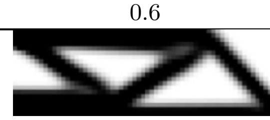


Fig. 8. Design domain with working conditions of MBB-beam.

Table 4. **Numerical example two's optimal topologies of different volume fractions**

Volume fraction f	0.4	0.5	0.6
SIMP method			
The proposed method			

Tables 4 and 5 display the optimal topologies and the corresponding results by using different interpolation methods in the optimization, respectively. From Table 4, one can see that the proposed method can lead to better topological designs with distinct boundaries which is greatly beneficial to designers in rationally extracting black-white boundaries without over- or under-estimations. Table 5

Table 5. **Numerical example two's comparison of the optimization results**

Volume fraction f		Compliance C	Gray-scale elements ($0.2 \leq x_i \leq 0.8$)
0.4	SIMP method	267.2560	221
	The proposed method	255.8830	36
0.5	SIMP method	211.0851	209
	The proposed method	207.5821	45
0.6	SIMP method	185.7014	223
	The proposed method	183.4463	55

indicates that when the proposed approach is employed to solve the topology optimization problems of continua, optimal topologies with lower compliance (or higher stiffness) and less number of gray-scale elements (relative densities between 0.2 and 0.8) will be obtained, other things being equal.

V. DISCUSSION AND CONCLUSIONS

The unique characteristic of the proposed methodology is that it can create topological designs with distinct boundaries with materials either close to 0 (void) or 1 (solid), besides the merit of guiding optimal topologies with lower compliance (or higher stiffness) in the topology optimization of continuum structures. From the numerical results, it can be found that the modified material interpolation method can accurately lead to optimal topology, and the gray-scale elements with intermediate material densities are effectively suppressed in the final designs in conjunction with the modified sensitivity filtering method. Plenty of numerical examples show that the effect is better when the penal factor a is between 6.5 and 9.0, while the density boundary factor m is between 1.5 and 2.5.

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