MULTIDISCIPLINARY DESIGN OPTIMIZATION BASED ON ALMABC ALGORITHM*

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Abstract: Multidisciplinary Design Optimization (MDO) is widely recognized as the key technology to the engineering design. However, due to the computational complexity of MDO, it is hard to find a suitable general algorithm for all MDO problems. At present, the calculation methods of MDO problems mainly have two types: traditional optimization methods and modern intelligent optimization algorithms. Traditional optimization methods have many inherent defects, such as difficult to solve discontinuous functions and easy to fall into local optimum. Therefore, in this paper we make contributions by proposing an improved Artificial Bee Colony (ABC) to solve the MDO problem. Firstly, we discuss the application of intelligent algorithms in MDO, and then the characteristics and superiorities of ABC algorithm are described. Secondly, an improved ABC algorithm based on Augmented Lagrange Multiplier (ALM) method is proposed. Finally, a mathematical example is provided to illustrate feasibility and validity of the proposed method.

Key words: Artificial Bee Colony; Augmented Lagrange Multiplier; Multidisciplinary Design Optimization.

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* This work is supported by the National Natural Science Foundation of China under the contract number 51475082.

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1. Introduction

Multidisciplinary Design Optimization (MDO) in the design of complex mechanical systems has been widely cognized, it is the key technology of advanced engineering design in the future [1-2]. This is because the MDO can put many designs of complex systems into a single composite design at the same time. It can significantly shorten the design lifecycle. Therefore, it is the main development trend of modern complex mechanical system designs [3-5]. At present, the calculation methods of MDO problems mainly have two types: traditional optimization methods and modern intelligent optimization algorithms. In some cases, traditional optimization methods can't solve the engineering optimization of modern complex mechanical systems. Such as the traditional gradient descent method, it requires that the objective functions are continuous and constrained area can be explicitly expressed. But in the actual engineering problems, the objective functions are often discontinuous, non-differentiable, even can't be expressed explicitly. Feasible regions are not connected. For modern intelligent algorithm [6-11], mathematical expressions are more relaxed, gradient information is not needed and the design space can be disconnected. In general, modern intelligent algorithm has many advantages such as good performance on global searching and avoiding falling into local optimum.

Some scholars use the intelligent algorithms to solve MDO problems. Prabhat [12-13] summarized genetic algorithm (GA), simulated annealing (SA) in the field of aircraft design optimization. Venter, G and Sobieski [14] applied PSO method to multidisciplinary design optimization of aircraft wing. Jenn-Long [15] etc. combined SA algorithm with factor analysis method to put forwards a new kind of Taguchi-SA algorithm, and applied it to the wing aerodynamic shape design optimization. Kazuhisa [16] hybridized PSO and GA methods to solve wing design problem. Christopher Gregory Hart [17] used target cascaded strategy and PSO as optimization solver to solve MDO concept ship design at the system level and subsystem level. This paper proposes an improved Artificial Bee Colony algorithm by combining augmented Lagrange multiplier method to solve the optimization problems with constraints.

2. Basic Artificial Bee Colony Algorithm

The basic ABC algorithm is a kind of high-performance algorithm based on swarm intelligence. The basic ABC algorithm was firstly proposed by Professor Karaboga in the literature [18] and has received extensive attention to scholars because of its advantages, such as labor division and cooperation, information sharing and self-organization. The basic ABC algorithm has broad application prospects since the structure of ABC algorithm is simple and its concepts are clear, the corresponding programming is easy to implement. Furthermore, it has excellent global optimization performance.

The objective function $f(X_i)$ needs to be conversed to fitness function:

$$fitness_i = \begin{cases} 1/(1+f(X_i)) & if\ f(X_i) \geq 0 \\ 1+abs(f(X_i)) & if\ f(X_i) < 0 \end{cases} \tag{2-1}$$

In formula (2-1), fitnessi is the value of fitness function which represents the number of the i-th food source of honey, so fitnessi cannot be negative.

In the process of exploiting a food source, a better solution V_i which is close to the original location X_i should be obtained. Assume that original location of D-dimensional design space is X_i=[$x_{i,1}$,…, $x_{i,D}$], a new V_i can be simply calculated by the following equations:

$$V_i = [x_{i,1}, \cdots, v_{i,j}, \cdots, x_{i,D}]$$

$$v_{i,j} = x_{i,j} + \varphi_{i,j}(x_{i,j} - x_{k,j}) \tag{2-2}$$

where $j \in \{1,2,\ldots,D\}$, $k \in \{1,2,\ldots,N_e\}$, $k \neq i$, N_e is the total number of food source, φ_{ij} is a random number which is generated from[-1,1]. When a food source is retried many times and the value of fitness function still can't be improved, employed bees will throw away this food source. A maximum of the retry count is the *Limit* value which is a key parameter in ABC algorithm.

3. Augmented Lagrange Multiplier Artificial Bee Colony -MDO (ALM ABC-MDO)

Although it has many advantages, the basic ABC has some deficiencies, such as slow convergence speed at later evolution process, weak local search ability, can't solve the optimization problems with constraints and etc. This paper proposes an improved Artificial Bee Colony algorithm by combining augmented Lagrange multiplier method to solve the optimization problems with constraints.

3.1. *Augmented Lagrange Multiplier Method*

For an optimization problem with equality and inequality constrains, augmented Lagrange equation can be given as follows:

$$ALM(X,\lambda,\beta,r_p) = f(X) + \sum_{j=1}^{r} \lambda_j h_j(X) + \sum_{i=1}^{m} \beta_i \alpha_i + r_p \sum_{j=1}^{r} h_j^2(X) + r_p \sum_{j=1}^{r} \alpha_i^2 \tag{3-1}$$

where $f(X)$ is objective function, $h_j(X)$ is j-th equation constraint, λ_j and β_i are Lagrange multipliers, respectively. the r_p is the fixed penalty parameter, α_i can be calculated by following formula:

$$\alpha_i = \max\{g_i(X), -\beta_i/(2r_p)\} \tag{3-2}$$

where $g_i(X)$ is i-th inequation. Lagrange multipliers can be calculated by:

$$\lambda_j^{(k+1)} = \lambda_j^{(k)} + 2r_p h_j(X) \tag{3-3}$$

$$\beta_i^{(k+1)} = \beta_i^{(k)} + 2r_p \max\{g_i(X), -\beta_i/(2r_p)\} \tag{3-4}$$

3.2. *ALMABC-MDO*

The detailed process of improved ABC algorithm is as follows:

Step1: Initialize ABC algorithm parameters: S_n is the number of bees (including the numbers of employed bees N_e and onlooker bees N_o), D is the search space dimension, *Limit* is the maximum number of retries, set the initial retry *trail*=0. *Maxcycle* is the maximum number of iterations.
Step2: According to the practical MDO problem construct augmented Lagrangian equation, set the initial Lagrange multipliers $\lambda^{(0)}{}_j=0$, $\beta^{(0)}{}_j=0$, $r_p=1$.
Step3: Initialize the locations of food sources, and calculate the corresponding fitness values.

for i=1 to N_e ***do***

for j=1 to D ***do***

generate each component of location:

$$x_{i,j} = x_{\min,j} + rand(0,1)(x_{\max,j} - x_{\min,j}) \tag{3-5}$$

end *for*

$trail_i$:=0; calculate $fitness_i$

end *for*

Step4: Let cycle=cycle+1, implement next iteration.
Step5: Perform employed bees, exploit each food source in turn.

for i=1 to N_e ***do***

According to the equation (2-2) generate a new location V_i.

Calculate the value of fitness $fitness(V_i)$.

if $fitness(V_i) > fitness_i$ **then** $X_i := V_i$; $fitness_i = fitness(V_i)$; $trail_i$=0

else

$trail_i := trail_i + 1$

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end ***if***

end ***for***

Step6: For each food source, calculate corresponding probability p_i according to the following formula:

$$p_i = fitness_i \Big/ \sum_{i=1}^{N_e} fitness_i \tag{3-6}$$

Step7: Perform onlooker bees. According to the probability p_i of each food source, implement roulette algorithm to choose a food source k, onlooker bee j exploit food source k.

Step8: Perform scout bees. Check each $trail_i$, if $trail_i \geq Limit$, the i-th employed bee will become scout bee and its position vector X_i will be regenerated according to (3-5), its fitness value is also recalculated, and reset $trail_i = 0$.

Step9: Compare fitness value $fitness_i$ of all food sources, find the optimal value.

Step10: If the cycle<*Maxcycle*, and the optimal value does not meet the given requirements, update λ'_i, β'_i, respectively, then go to Step 4.

Step11: Otherwise, terminate iteration process, output current optimal value.

4. Case Studies

The following mathematical example is a classic MDO problem which has two subsystems. Each subsystem has a coupling state variable. Nonlinear coupling relationship exists between the two subsystems [19]. The mathematical expression is as follows:

$$\begin{aligned}
Min \quad & f = x_1^2 + x_2 + y_1 + e^{-y_2} \\
& y_1 = x_1 + x_2 + x_3^2 - 0.2y_2 \\
& y_2 = \sqrt{y_1} + x_2 + x_3 \\
s.t. \quad & g_1 = \left(\frac{y_1}{3.16}\right) - 1 \geq 0;\ g_2 = 1 - \left(\frac{y_2}{24}\right) \geq 0 \\
& h_1 = x_1 + x_2 + x_3^2 - 0.2y_2 - y_1 \\
& h_2 = \sqrt{y_1} + x_2 + x_3 - y_2 \\
& 0 \leq x_1 \leq 10; 0 \leq x_2 \leq 10; -10 \leq x_3 \leq 10
\end{aligned}$$

The mathematical model has two equality constraints and two inequality constraints. According to formula (3-1), construct the augmented Lagrangian equation:

$$ALMf(X,\lambda,\beta,r_p) = f(X) + \sum_{j=1}^{2} \lambda_j h_j + \sum_{i=1}^{2} \beta_i \alpha_i + r_p \sum_{j=1}^{2} h_j^2 + \sum_{j=1}^{2} \alpha_i^2$$
$$= x_1^2 + x_2 + y_1 + e^{-y_2} + \lambda_1(x_1 + x_2 + x_3^2 - 0.2y_2 - y_1) + \lambda_2(\sqrt{y_1} + x_2 + x_3 - y_2)$$
$$+ \beta_1 \max\{g_1, -\beta_1/(2r_p)\} + \beta_2 \max\{g_2, -\beta_2/(2r_p)\} + r_p((x_1 + x_2 + x_3^2 - 0.2y_2 - y_1)^2$$
$$+ (\sqrt{y_1} + x_2 + x_3 - y_2)^2) + (\max\{g_1, -\beta_1/(2r_p)\})^2 + (\max\{g_2, -\beta_2/(2r_p)\})^2$$

In order to reduce the accidental error, run the ALMABC program 10 times; then take the average value as the final result of objective function *f* which is shown in Table 1. The value 0.0044 is the standard deviation. Three methods listed in the table, the method 1 is the optimization results in reference [19], the method 2 is sequential quadratic programming (SQP) method, and method 3 is our proposed method. The convergence curve of objective function is as shown in Figure 1. Through the optimization results we can see that three methods all obtain almost the same values of objective function *f*, which demonstrates the feasibility of our proposed ALMABC method. Furthermore, compared to the existing MDO methods, our proposed method is very simple and highly flexible. However, we also can see that in the later iteration period, the local optimization search ability is poor and convergence speed is slow.

Table 1. Optimization results

	x_1	x_2	x_3	y_1	y_2	f
Method 1	0	0	1.9776	3.16	3.7556	3.1835
Method 2	0	0	1.9776	3.16	3.7553	3.1834
Method 3	0.01073	0	1.9715	3.16	3.7406	3.1839(0.0044)

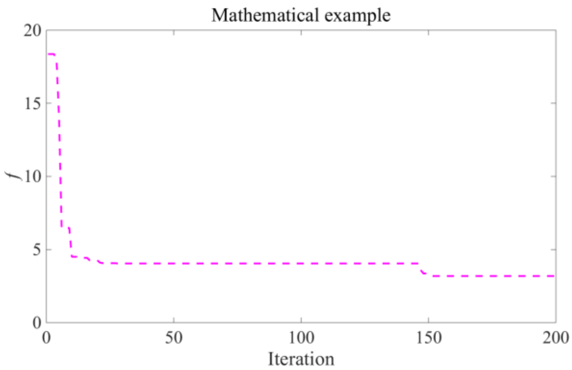

Figure 1. The convergence curve of objective function

5. Summary and Outlook

(1) In this paper, an improved ALMABC algorithm based on augmented Lagrange equation is proposed. A mathematical example is provided to illustrate the application and reliability of the proposed method.

(2) The proposed ALMABC algorithm still has its own limitations. No matter in mathematical example or engineering optimization design problem, the local optimization search ability is poor in the later iterations and convergence speed is slow. The future work will focus on combining the target cascading method or PSO algorithm with ALMABC to improve the precision and efficiency.

Acknowledgments

This research is supported by the National Natural Science Foundation of China under the contract number 51475082.

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