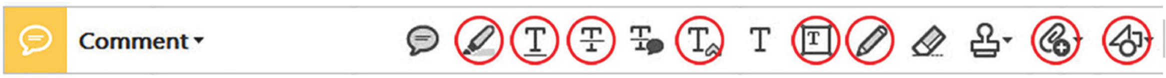
Page Proof Instructions and Queries

Journal Title: ADE
Article Number: 680177

Greetings, and thank you for publishing with SAGE. We have prepared this page proof for your review. Please respond to each of the below queries by digitally marking this PDF using Adobe Reader (free at https://get.adobe.com/reader).

Please use *only* the circled tools to indicate your requests and responses, as edits via other tools/methods are not compatible with our software. To ask a question or request a formatting change (such as italics), please click the tool and then choose "Text Callout." To access the necessary tools, choose "Comment" from the right-side menu.



Sl. No.	Query
	GQ: Please check if the correct license type (CC-BY-NC/CC-BY/CC-BY-NC-ND) appears on your article and matches with what you had specified in the license agreement form.
	GQ: Please confirm that all author information, including names, affiliations, sequence, and contact details, is correct.
	GQ: Please review the entire document for typographical errors, mathematical errors, and any other necessary corrections; check headings, tables, and figures.
	GQ: Please ensure that you have obtained and enclosed all necessary permissions for the reproduction of artworks (e.g. illustrations, photographs, charts, maps, other visual material, etc.) not owned by yourself. please refer to your publishing agreement for further information.
	GQ: Please note that this proof represents your final opportunity to review your article prior to publication, so please do send all of your changes now.
	GQ: Please confirm that the funding and conflict of interest statements are accurate.
1	AQ: Please provide expansions for 'FORM' and 'SORM' if appropriate.
2	AQ: Please provide an expansion for 'RBTV' if appropriate.
3	AQ: Please provide an expansion for 'TIV' if appropriate.
4	AQ: Please provide publisher details for Ref. 20.
5	AQ: Please provide location, date and month of conference for Ref. 21.
6	AQ: Please provide publisher details for Ref. 34.

Special Issue Article

Advances in Mechanical Engineering

Advances in Mechanical Engineering
2016, Vol. 8(11) 1–11
© The Author(s) 2016
DOI: 10.1177/1687814016680177
aime.sagepub.com
SAGE

Multidisciplinary reliability design optimization under time-varying uncertainties

Huanwei Xu, Wei Li, Xin Wang, Cong Hu and Suichuan Zhang

Abstract

Degradation failure is one of the main reasons for complex mechanical systems losing their functions. Research on multidisciplinary design optimization under uncertainties should shift from static uncertainties to time-varying uncertainties. Aiming at time-varying uncertainties in mechanical systems, we put forward a multidisciplinary reliability design optimization method using stochastic process theory. First, we investigated the characteristics of time-varying uncertainties in complex mechanical systems, and then utilized stochastic process theory to quantify time-varying uncertainties. Second, through combining the multidisciplinary simultaneous analysis and design optimization method, the model of multidisciplinary design optimization under time-varying uncertainties is established. Moreover, a mathematical problem and an engineering example are provided to illustrate the accuracy and effectiveness of the proposed method.

Keywords

Multidisciplinary design optimization, time-varying uncertainty, stochastic process, time-varying reliability model, simultaneous analysis and design

Date received: 5 August 2016; accepted: 10 October 2016

Academic Editor: Yongming Liu

Introduction

Multidisciplinary design optimization (MDO) of complex mechanical systems has shown wide application recently.[1–4] Increasing attentions are being paid on MDO under uncertainties in recent years. Du and Chen[5,6] proposed an uncertainty calculation method based on global and local sensitivity equation by combining uncertainty analysis method with error propagation in the process of physical measurement, and then put forward a system uncertainty analysis (SUA) method and concurrent subsystem uncertainty analysis (CSSUA) method. Gu et al.[7] derived limit expression of uncertainty propagation based on Taylor series method, and then built stochastic uncertainty analysis model. The above-mentioned researches have made tremendous contributions on MDO under aleatory and epistemic uncertainties. However, note from researches in previous studies[8–13] that the degradation failure is one of the main reasons for complex mechanical systems lost their functions. There are variant time-varying uncertainties in complex mechanical systems, such as performance degradation of electronic component, strength decrease, material aging, wear, oxidation, and corrosion.[14,15] Although research methods to deal with aleatory and epistemic uncertainties in MDO may largely improve the reliability of complex systems, however, these research methods cannot accurately describe the sources, essential characteristics, and propagation properties of time-varying uncertainties and

School of Mechatronics Engineering, University of Electronic Science and Technology of China, Chengdu, P.R. China

Corresponding author:
Huanwei Xu, School of Mechatronics Engineering, University of Electronic Science and Technology of China, Chengdu 611731, Sichuan, P.R. China.
Email: hwxu@uestc.edu.cn

cannot simply be applied to deal with time-varying uncertainties in MDO.

Currently, methods to deal with time-varying uncertainties under single-discipline analysis can be divided into two categories. (1) Probabilistic reliability methods, such as Bayesian method and Monte Carlo method. These methods are very effective when dealing with random time-varying uncertainties. However, these methods need to know the probability distributions in advance, require that time is clearly defined, and samples must be adequate. In addition, these prerequisites are often difficult to obtain in the design process. Melchers[16] indicate that when there is only one random load, the force can be expressed by the extreme value distribution, and then calculate structure failure probability using either FORM or SORM method, another case is that using series reliability method to calculate the reliability of the life cycle by only considering some of the key period in life cycle, such as storm AQ1 periods. Veneziano et al.[17] assumed that time-varying uncertainties are Gauss distributions and proposed a system reliability analysis method. Breitung and Rackwitz[18] proposed a system reliability analysis method using rectangular wave renewal process to describe time-varying uncertainties, and combined Gauss process and rectangular wave renewal process to deal with time-varying uncertainties.[19–21] Li and Zhang[22] assumed that the deterioration process of resistance was a Gamma process and proposed a time-variant reliability assessment. (2) Non-probabilistic reliability methods, such as interval analysis and convex sets. These methods do not take full advantage of the information in design process and the final design results are conservative. According to the upper and lower boundary of time-interval reliability, Shinozuka[23] provided calculation formulas. Andrieu-Renaud et al.,[24] Cazuguel et al.,[25] and Li and Mourelatos[26] established the formulas to calculate the time-varying reliability index and compared the differences and relations between time interval reliability and moment reliability. Jiang et al.[27] proposed an effective non-probabilistic model process method for time-variant uncertainty analysis.

Until now, the methods to process time-varying uncertainties under single-discipline analysis have made certain progress. However, during the whole product life cycle, forms of time-varying uncertainties are diversified and often correlated to each other. Since there are hierarchical and non-hierarchical hybrid coupled relationships between subsystems in MDO, time-varying uncertainties which exist in subsystem levels may have different influence on the final output of the system. In addition, MDO is a kind of coordination system optimization method for large systems, which leads to difficulties to quantify time-varying uncertainties.

Aiming at dealing with time-varying uncertainties in MDO, this article proposes a multidisciplinary reliability design optimization (MRDO) method under time-varying uncertainties. The remainder of this article is arranged as follows. In section "Time-varying reliability model," we analyze the characteristics of time-varying uncertainties, and then introduce stochastic Ito process to quantify time-varying uncertainties. In section "Time-varying reliability MDO," MDO model under time-varying uncertainties is established, and the specific operation steps are given. In section "Case studies," a mathematical problem and an engineering example are given to illustrate the feasibility and effectiveness of the proposed method. Section "Conclusion" gives the research conclusion.

Time-varying reliability model

Reliability analysis under time-varying uncertainties

For mechanical products, time-varying uncertainties include material performance, working environment,

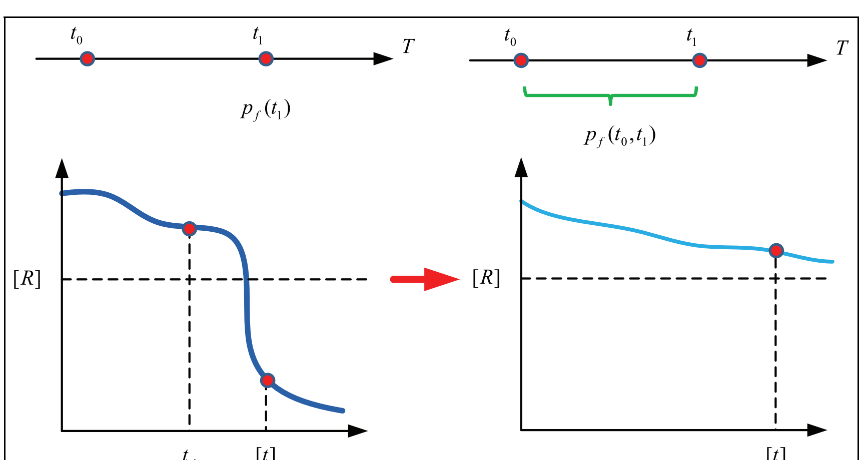


Figure 1. Time-varying reliability diagram.

service time, and load effect.[14,15] The product performance degrades gradually over time. The degradation process of product performance is a dynamic time-varying process.[15] Time-varying reliability method is often applied to deal with time-varying uncertainties. Time-varying reliability means that the failure rate p_f (t_0–t_1) of limit state function is less than the given failure rate during time interval t_0–t_1. In other words, the reliability R of limit state function at any time is greater than the allowable reliability [R], which is shown in Figure 1.

Currently, the methods considering time-varying uncertainties assume that strength and stress of mechanical parts follow certain probability distributions, respectively,[28–31] which cannot be used to evaluate dynamic evolution process of time-varying uncertainties. According to this, we attempt to propose a time-varying reliability prediction model based on stochastic differential equation theory which can calculate the reliability index at any time.

Stochastic differential equation

Compared to traditional probability methods, Ito differential equation is better to deal with uncertainties in mechanical systems. The structure of Ito differential equation is simple, theoretical concept is clear, so it is very easy to combine Ito differential equation with MDO to solve time-varying uncertainties. Assume $\Omega = \{\omega\}$ is the sample space of stochastic experiment, and T is a set of parameters (time). For each $t \in T$, there is a random variable $X(t, \omega)$. A set of variables $X(t, \omega)$ is a stochastic process. The definition domain of ω is the entire sample space, and the definition domain of t is the entire timeline [0, ∞] or a time period [0, T][32]

$$X(t, \omega) : \{\Omega\} \times [0, T] \to R$$

Assume that there are m time-varying factors, then

$$X(t, \omega) = [x_1(t, \omega), x_2(t, \omega), \ldots, x_m(t, \omega)]^T$$

Set (Ω, F, P) is a probability space; F is the σ-algebra of Ω, P is the probability measure of Ω, and the random variable $X(t, \omega)$ should satisfy the following Ito equation

$$\begin{cases} dX(t, \omega) = u(t)X(t, \omega)dt + v(t)X(t, \omega)dW(t, \omega) \\ X(t_0) = X_0 \end{cases} \tag{1}$$

where $u(t)$ is the drift rate which reflects the influence of deterministic factors, and $v(t)$ is the fluctuation rate which reflects the influence of time-varying uncertainties.

The drift rate can be expressed as follows

$$u(t) = \frac{1}{n-1} \sum_{1}^{n} \ln \frac{X_{i+1}}{X_i} \tag{2}$$

where X_i is the observation value at time point i (i = 1, 2, …, n).

The fluctuation rate can be expressed as

$$v(t) = \left[\sum_{1}^{n} \frac{\left(\ln \frac{X_{j+1}}{X_j} - u(t) \right)^2}{n-1} \right]^{1/2} \tag{3}$$

where X_j is the observation at time point j (j = 1, 2, …, n). The draft rate and the fluctuation rate are affected by the number of observation points. The more the observation points, the more the precision of the result. For different problems, the number of observation points can be set according to precision needed. Although Ito differential equation has many advantages, it is not applicable for those cases when load mutation exists in the engineering structure.

Time-varying reliability model

If the random variable X satisfies the following equation

$$dX(t, \omega) = u(t)X(t, \omega)dt + v(t)X(t, \omega)dW(t, \omega) \tag{4}$$

equation (4) also can be expressed as

$$\frac{dX(t, \omega)}{X(t, \omega)} = u(t)dt + v(t)dW(t, \omega) \tag{5}$$

Assume that $Y(t) = \ln X(t, \omega)$, which leads to

$$\ln X(t) - \ln X(0) = \int_0^t \frac{dX}{X} + \frac{1}{2} \int_0^t \frac{-1}{X^2} v^2 X^2 ds \tag{6}$$

where s is the time variable

$$Y(t) - Y(0) = \int_0^t v dW_s + \int_0^t (u - 0.5v^2) ds$$

According to equations (7) and (8)

$$\begin{gathered} \int_0^t ds = t, \quad \int_0^t dW_s = W_t - W_0, \; W_0 = 0 \\ Y(t) = \ln X(0) + \left(u - \frac{1}{2} v^2 \right) t + vW(t) \end{gathered} \tag{7}$$

$$E(\ln X(t)) = \ln X(0) + \left(u - \frac{1}{2} v^2 \right) t \tag{8}$$

We can obtain

$$\begin{aligned}\ln E(X(t)) &= \ln\left[E(X(0))e^{\left(u-\frac{1}{2}v^2\right)t\,+\,vW(t)}\right]\\ &= \ln X(0) + \left(u-\frac{1}{2}v^2\right)t + \ln e^{0.5v\sqrt{t}}\end{aligned} \tag{9}$$

The relationship between the expected logarithm and logarithmic expectation can be obtained by

$$\ln E(X(t)) = \frac{E(\ln X(t)) + D(\ln X(t))}{2} \tag{10}$$

By substituting equation (10) in equations (8) and (9), we can obtain

$$D(\ln X(t)) = v\sqrt{t}$$

where ln $X(t)$ is a normal distribution function, and the corresponding mean value and variation coefficient are $\ln X(0) + (u - v^2/2)t$ and $v\sqrt{t}$, respectively

$$\ln X(t) \sim N\left[\ln X(0) + \left(u - \frac{1}{2}v^2\right)t, v^2 t\right] \tag{11}$$

Assume that the limit state function is

$$G(t, \omega) = \mathrm{S}(t) - \delta(t) \tag{12}$$

where $G(t, \omega) > 0$ means the structure is safe, and $G(t, \omega) < 0$ indicates failure of the structure. Therefore, the reliability probability is expressed as

$$R(t) = P(\ln S(t) - \ln\delta(t) > 0)$$

Let $Z = \ln S(t) - \ln\delta(t)$, and ln $S(t)$ and ln $\delta(t)$ are independent from each other. Thus, Z also obeys normal distribution. The mean value and variance of Z are expressed as

$$u_Z = u_{\ln S(t)} - u_{\ln\delta(t)} \tag{13}$$

$$\sigma_Z = \sqrt{\sigma_{\ln S(t)} + \sigma_{\ln\delta(t)}} \tag{14}$$

Then, the probability density function of Z is

$$\phi(Z) = \frac{1}{\sqrt{2\pi}\sigma_Z} e^{\frac{-(Z-u_Z)^2}{2\sigma_Z^2}} \tag{15}$$

Let $s = (Z - u_Z)/\sigma_Z$. Since reliability index can be expressed as

$$\begin{aligned}R(t) = P(Z>0) &= \int_0^\infty \phi(Z)dZ = \sqrt{2\pi}\int_{Z_{R(t)}}^\infty e^{-0.5s^2} ds\\ &= \sqrt{2\pi}\int_{-\infty}^{-Z_{R(t)}} e^{-0.5s^2} ds\end{aligned}$$

The reliability index can be obtained by $R(t) = \Phi(-Z_{R(t)})$. Substitute equation (15) in equations (13) and (14)

$$Z_{R(t)} = -\frac{u_{\ln S(t)} - u_{\ln\delta(t)}}{\sqrt{\sigma_{\ln S(t)} + \sigma_{\ln\delta(t)}}} \tag{16}$$

where

$$\begin{aligned}u_{\ln S(t)} - u_{\ln\delta(t)} = {} & \ln S(0) + (u_S - 0.5v_S^2)t - (\ln\delta(0)\\ & + (u_\delta - 0.5v_\delta^2)t)\end{aligned}$$

and

$$\begin{aligned}&\ln S(0) - \ln\delta(0) = -Z_{R(t)}\sqrt{\sigma_{\ln S(t)} + \sigma_{\ln\delta(t)}}\\ &+ ((u_S - 0.5v_S^2) - (u_\delta - 0.5v_\delta^2))t\frac{S(0)}{\delta(0)}\\ &= \exp\left\{-Z_{R(t)}\sqrt{\sigma_{\ln S(t)}^2 + \sigma_{\ln\sigma(t)}^2}\right.\\ &\left.-\left[\left(u_s - \frac{1}{2}v_s^2\right) - \left(u_\delta - \frac{1}{2}v_\delta^2\right)\right]t\right\}\end{aligned} \tag{17}$$

Once the reliability index $Z_{R(t)}$ is given, take initial $S(0)$ and $\delta(0)$ as constrains to quantify time-varying uncertainties, and the time-varying reliability optimization model can be constructed.

Time-varying reliability MDO

Multidisciplinary simultaneous analysis and design

Generally, a multidisciplinary optimization problem consists of two subsystems. The multidisciplinary simultaneous analysis and design (SAND) method[33] is introduced to construct the optimization framework, which is shown in Figure 2. Since time-varying uncertainties analysis is complicated and time-consuming, it is better to combine it with simple and easy MDO strategy. Compared to the other MDO strategies, SAND expression is simpler and easier to understand. SAND strategy can substitute solver for analyzer, which can reduce the high time-consuming analysis process.

In Figure 2, X_s and P_s are the shared design variable vector and shared design parameter vector, respectively.

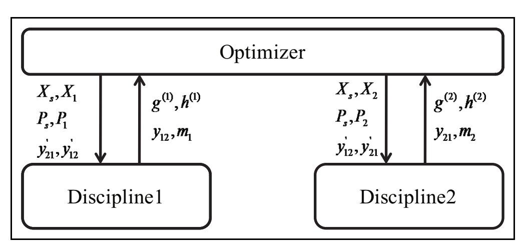


Figure 2. SAND optimization framework.

X_i is the design variable vector of discipline i, P_i is the design parameter vector of discipline i, y_{ij} is the coupled state variable vector which is the output from i and input to discipline j, and y'_{ij} is the corresponding auxiliary variable vector. $g(i)$ and $h(i)$ represent the inequality and equality constraints of discipline i. m_1 and m_2 are the residuals of coupled state variable and auxiliary variable after the multidisciplinary analysis. SAND optimization strategy passes the values of design variable vector, coupled state variable vector, and auxiliary variable vector to each discipline; according to the values of the design variable vector and auxiliary variable vector, each discipline gets the values of output coupled variable vector after multidisciplinary analysis. At the same time, coupled state variable vector, residuals, and constraints of all disciplines are passed to the optimizer. Using this optimization strategy, each discipline can be analyzed independently. The optimization model of SAND is listed as follows

$$\begin{aligned} \min \quad & f(X_s, P_s, X_i, P_i, Y) \\ s.t. \quad & g^{(i)}(X_s, P_s, X_i, P_i, Y'_{\bullet i}) \le 0 \\ & h^{(i)}(X_s, P_s, X_i, P_i, Y'_{\bullet i}) = 0 \\ & m_i = Y_{i\bullet} - Y'_{\bullet i} = 0\ ;\ i = 1,2 \\ & Y = \{y_{12}, y_{21}\}\ ;\ Y' = \{Y'_{12}, y^1_{21}\} \\ DV = & \{X_s, X_i, Y, Y'\} \end{aligned} \tag{18}$$

Ito-RBTV-MDO model

The time-varying reliability optimization model of MDO is shown as follows

$$\begin{aligned} \min \quad & f(X_{s,d}, \bar{X}_{s,r}, X_{i,d}, \bar{X}_{i,r}, P_{s,d}, \bar{P}_{s,r}, P_{i,d}, \bar{P}_{i,r}, \bar{Y}) \\ s.t. \quad & \Pr[G^{(i)}_{TV}(X_{s,d}, X_{s,r}, X_{i,d}, X_{i,r}, P_{s,d}, P_{s,r}, P_{i,d}, P_{i,r}, Y_{\bullet i}) \ge 0] \\ & \ge [R^{TV}_i] \\ & g^{(i)}(X_{s,d}, \bar{X}_{s,r}, X_{i,d}, \bar{X}_{i,r}, P_{s,d}, \bar{P}_{s,r}, P_{i,d}, \bar{P}_{i,r}, \bar{Y}_{\bullet i}) \le 0 \\ & h^{(i)}(X_{s,d}, \bar{X}_{s,r}, X_{i,d}, \bar{X}_{i,r}, P_{s,d}, \bar{P}_{s,r}, P_{i,d}, \bar{P}_{i,r}, \bar{Y}_{\bullet i}) = 0 \\ & X^l_{s,d} \le X_{s,d} \le X^u_{s,d}, X^l_{i,d} \le X_{i,d} \le X^u_{i,d}, \\ & \bar{X}^l_{s,r} \le \bar{X}_{s,r} \le \bar{X}^u_{s,r}, \bar{X}^l_{i,r} \le \bar{X}_{i,r} \le \bar{X}^u_{i,r} \\ DV = & \{X_{s,d}, \bar{X}_{s,r}, X_{i,d}, \bar{X}_{i,r}\} \end{aligned} \tag{19}$$

where $X_{s,d}$ and $P_{s,d}$ are the shared deterministic design variable vector and shared deterministic design parameter vector, respectively. $X_{i,d}$ and $P_{i,d}$ are the shared deterministic design variable vector and shared deterministic design parameter vector of discipline i, respectively. $X_{s,r}$ and $P_{s,r}$ are the shared random variable vector and shared random parameter vector, respectively. $X_{i,r}$ and $P_{i,r}$ are the shared random variable vector and shared random parameter vector of discipline i, respectively. $\bar{X}$ is the mean value of X. $G^{(i)}_{TV}(\bullet)$ is the limit state function by considering time-varying uncertainties. $\Pr[G^{(i)}_{TV}(\cdot)>0] \ge [R_i]$ is the time-varying reliability constraint of discipline i, and $[R^{TV}_i]$ is the reliability requirement of discipline i. AQ2

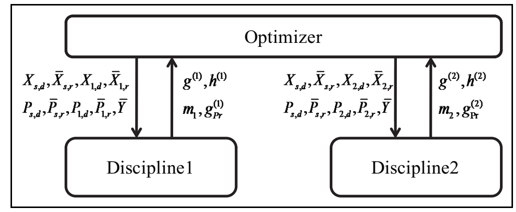


Figure 3. Ito-RBTV-MDO optimization framework.

The two subsystems Ito-RBTV-MDO model is shown in Figure 3.

In Figure 3, $g^{(i)}_{\Pr}$ is the time-varying reliability constraint equation of discipline i. The optimization process is given as follows:

Step 1. Deterministic MDO is implemented using the SAND method.
Step 2. Time-varying uncertainties are analyzed and quantified (the data of time-varying uncertainties are obtained by simulate sampling method in this article).
Step 3. Apply Ito equation theory to construct reliability optimization models under time-varying uncertainties.
Step 4. Convert time-varying reliability optimization models to constraints in MDO and implement a new deterministic MDO.

The corresponding flow chart is shown in Figure 4.

Case studies

In this section, both a mathematical example and a case study of an engineering system are introduced to illustrate feasibility and validity of the proposed method.

A mathematical example

This is a very classic MDO problem which includes two disciplines. Each discipline has a coupled state variable; nonlinear coupled relationship exists between two disciplines.[34] In order to reflect actual situation more clearly, we have modified this mathematical example. Two time-varying uncertainties p_1 and p_2 are added in optimization model, which to research how system limit state function $G(t, \omega)$ are effected.

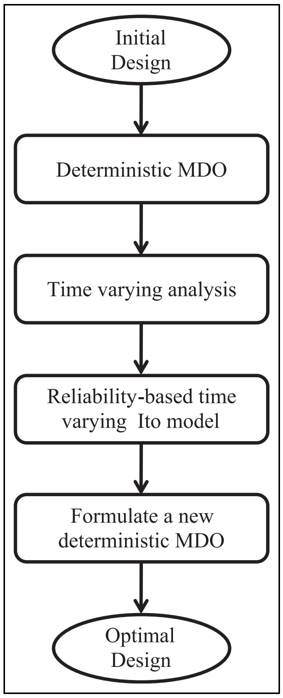


Figure 4. Time-varying reliability optimization flow chart.

The optimization model is expressed as follows

$$\begin{aligned} Min \quad & f = (x_1 - 1.2)^2 + (x_2 - 1.5)^2 + y_1 + e^{-y_2} - p_1 p_2 \\ s.t. \quad & g_1 = \left(\frac{y_1}{3.16}\right) - 1 \geq 0 \\ & g_2 = 1 - \left(\frac{y_2}{24}\right) \geq 0 \end{aligned}$$

where

$$\begin{aligned} y_1 &= x_1 + x_2 + x_3^2 - 0.2y_2 - p_1 p_2 \\ y_2 &= \sqrt{y_1} + x_2 + x_3 + p_1 p_2 \end{aligned}$$

The corresponding SAND model is shown in Figure 5.

The corresponding limit state function is given by $G(t, \omega) = (p_1(t, \omega) + 0.2)(p_2(t, \omega) + 0.4) - x_1 x_2 x_3$. In this mathematical example, we only consider how the system performance is affected by time-varying uncertainties. How to establish a reasonable and scientific limit state function in the practical engineering problems depends on the requirements of system performance in design stage.

The design parameters p_1 and p_2 are time-varying uncertainties. The corresponding sampling data are obtained by MATLAB simulation during 10 years, which is shown in Figure 6.

Using equations (3) and (4), to calculate the drift rates and fluctuation rates

$$\begin{aligned} u_{p_1} &= -0.000926\ ;\ v_{p_1} = 0.012 \\ u_{p_2} &= -0.002\ ;\ v_{p_1} = 0.024 \end{aligned}$$

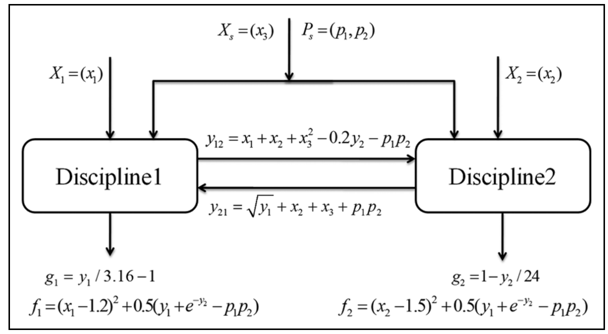


Figure 5. SAND model of mathematical example.

State function is expressed as

$$\begin{aligned} S(t) &= (p_1(t, \omega) + 0.2)(p_2(t, \omega) + 0.4) \\ \delta(t) &= x_1 x_2 x_3 \\ R(t) &= P(G(t, \omega) > 0) = P(\ln S(t) - \ln \delta(t) > 0) \end{aligned}$$

The design requirement of reliability index is 0.998 after 10 years, then

$$Z_{R(t)} = Z_{R(120)} = -2.8782$$

According to equation (17), we have

$$\frac{S(0)}{\delta(0)} = e^{0.8151} = 2.2594$$

Through updating state equation under time-varying uncertainties and implementing the optimization process mentioned in section "Ito-RBTV-MDO model," the corresponding optimization results are shown in Table 1, where TIV is the optimization result without considering time-varying uncertainties; RBTV is the reliability optimization result under time-varying uncertainties.

The reliability indexes of state function in each year are shown in Figure 7.

From the optimization results of these two methods, we can note that the value of objection function of RBTV method increases by 10.1% than TIVs. However, without considering the time-varying uncertainties, the reliability index of TIV method is 0.8262 after 10 years, which is obviously lower than the reliability index of design requirement 0.998. The reliability index of RBTV method is 0.9999 after 10 years by considering the time-varying uncertainties, which still satisfied the design requirement of reliability.

An engineering example

This is a speed-reducer optimization problem,[35] and the structure is shown in Figure 8. The objective function is to minimize the volume of the structure. The main constrains are the bending stress and contact stress of the

Table 1. Optimization results.

	x_1	x_2	x_3	y_1	y_2	f
TIV	1.200	1.500	1.978	3.160	7.256	−0.8393
RBTV	1.407	1.676	0.991	3.160	4.444	−0.7545

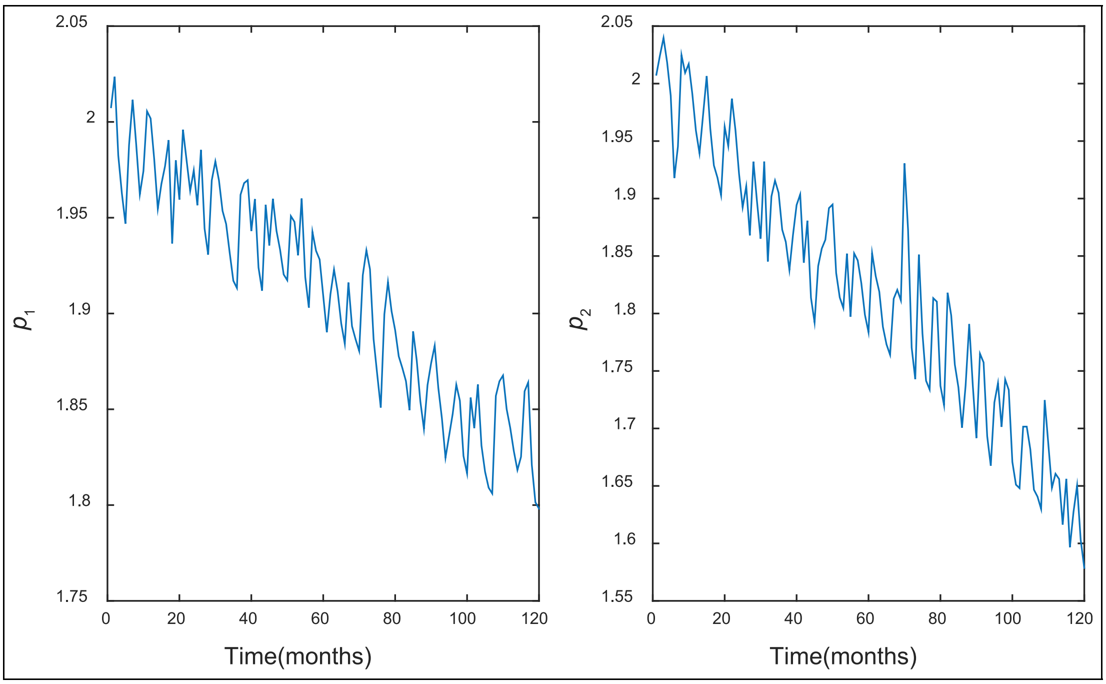


Figure 6. Time-varying data of two design parameters.

gear tooth, the torsion deformation, and stress requirements of the shafts.

The corresponding optimization model is as follows

$$
\begin{aligned}
Min\ f &= 0.7854x_1x_2^2(3.3333x_3^2 + 14.9334x_3 - 43.0934) \\
&\quad - 1.5079x_1(x_6^2 + x_7^2) \\
&\quad + 7.477(x_6^3 + x_7^3) + 0.7854(x_4x_6^2 + x_5x_7^2) \\
s.t.\quad g_1 &= 27/(x_1x_2^2x_3) - 1 \leq 0 \\
g_2 &= 397.5/(x_1x_2^2x_3^2) - 1 \leq 0 \\
g_3 &= 1.93x_4^2/(x_2x_3x_6^4) - 1 \leq 0 \\
g_4 &= 1.93x_5^2/(x_2x_3x_7^4) - 1 \leq 0 \\
g_5 &= 10\sqrt{(745x_4/(x_2x_3))^2 + 1.69 \times 10^7}/x_6^3 - 1100 \leq 0 \\
g_6 &= 10\sqrt{(745x_5/(x_2x_3))^2 + 1.575 \times 10^8}/x_7^3 - 850 \leq 0 \\
g_7 &= (1.5x_6 + 1.0)/x_4 - 1 \leq 0 \\
g_8 &= (1.1x_7 + 1.0)/x_5 - 1 \leq 0 \\
g_9 &= x_2x_3 - 40 \leq 0 \\
g_{10} &= 5 - x_1/x_2 \leq 0 \\
g_{11} &= x_1/x_2 - 12 \leq 0
\end{aligned}
$$

where x_1 is the tooth width; x_2 is the gear module; x_3 is the number of teeth of the pinion; x_4 is the distance between the bearings 1; x_5 is the distance between bearings 2; x_6 is the diameter of the shaft 1; x_7 is the diameter of the shaft 2; g_1 and g_2 are the constraints of bending stress and contact stress, respectively; g_3–g_8 are the constraints of the shaft deformation, stress, and so on; and g_9–g_{11} are the geometric constraints. The bounds of each design variable are shown in Table 2.

The limit state functions of g_5 and g_6 are as follows

$$
\begin{aligned}
G_1(t,\omega) &= 1100x_6^3 - 10\sqrt{(745x_4/(x_2x_3))^2 + 1.69 \times 10^7} \\
G_2(t,\omega) &= 850x_7^3 - 10\sqrt{(745x_5/(x_2x_3))^2 + 1.575 \times 10^8}
\end{aligned}
$$

Assuming that design variables x_6 and x_7 are time-varying uncertainties. The corresponding data of time-varying uncertainties are obtained by simulation methods, which are shown in Figure 9.

Using equations (3) and (4) to calculate the drift rate and fluctuation rate

Table 2. Lower and upper bounds of design variables (mm).

Design variables	x_1	x_2	x_3	x_4	x_5	x_6	x_7
Value range	2.6–3.6	0.7–0.8	17–28	7.3–8.3	7.3–8.3	2.9–3.9	5.0–5.5

Table 3. Optimization results.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	g_5	g_6	f
TIV	3.5	0.7	17	7.3	7.715	3.35	5.287	1.032E−8	0	2994.2
RBTV	3.5	0.7	18	7.3	7.716	3.35	5.287	−0.5142	−0.2345	3171.7

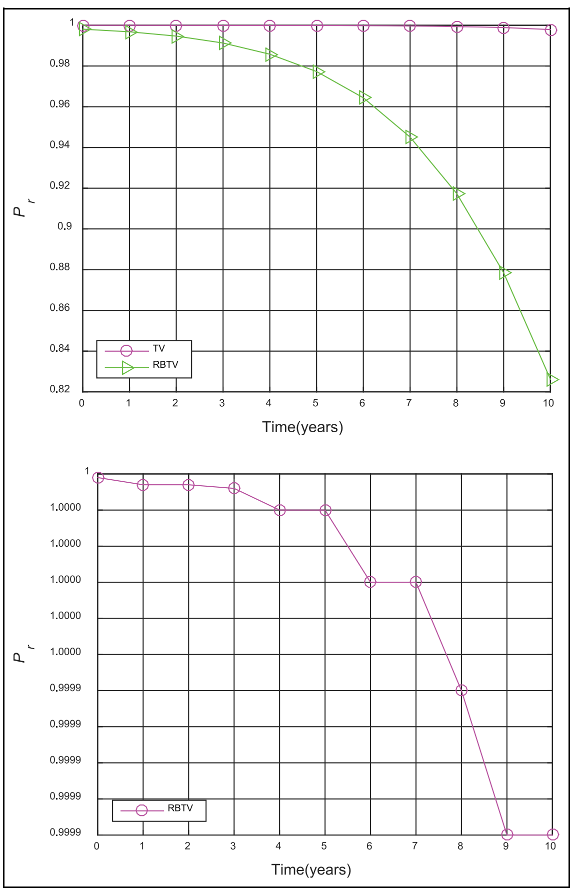


Figure 7. Comparison of reliability indexes.

$$u_{x_6} = -0.0001207\ ;\ v_{x_6} = 0.001092$$
$$u_{x_7} = -0.00007335\ ;\ v_{x_7} = 0.0005607$$

Let us consider

$$S_1(t) = 1100x_6^3,\ \delta_1(t) = 10\sqrt{(745x_4/(x_2x_3))^2 + 1.69 \times 10^7}$$
$$S_2(t) = 850x_7^3,\ \delta_2(t) = 10\sqrt{(745x_5/(x_2x_3))^2 + 1.575 \times 10^8}$$
$$R_1(t) = P(G_1(t,\omega)>0) = P(\ln S_1(t) - \ln \delta_1(t)>0)$$
$$R_2(t) = P(G_2(t,\omega)>0) = P(\ln S_2(t) - \ln \delta_2(t)>0)$$

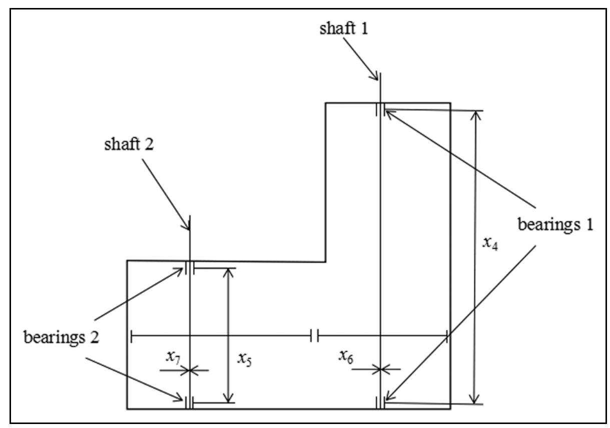


Figure 8. Structure diagram of speed reducer.

Suppose 1 year later, the requirement of reliability index is 0.98, then

$$Z_{R_1(t)} = Z_{R_2(t)} = Z_{R(12)} = -2.0538$$

According to equation (17)

$$\frac{S_1(0)}{\delta_1(0)} = e^{0.3932} = 1.4817$$
$$\frac{S_2(0)}{\delta_2(0)} = e^{0.2018} = 1.2236$$

The corresponding optimization results are shown in Table 3. TIV is the optimization result without considering time-varying uncertainties; RBTV is the reliability optimization result under time-varying uncertainties.

The reliability indexes of limit state functions in each month are shown in Figure 10.

Without considering the time-varying uncertainties, the reliability indexes of g_5 and g_6 are 0.4991 and 0.5010, respectively, at initial time. Considering the time-varying uncertainties, the reliability indexes of g_5 and g_6 are 0.9999 and 0.99140, respectively, at initial time. 1 year later, the reliability indexes of g_5 and g_6 are 0.9987 and 0.9813, which still meet the reliability

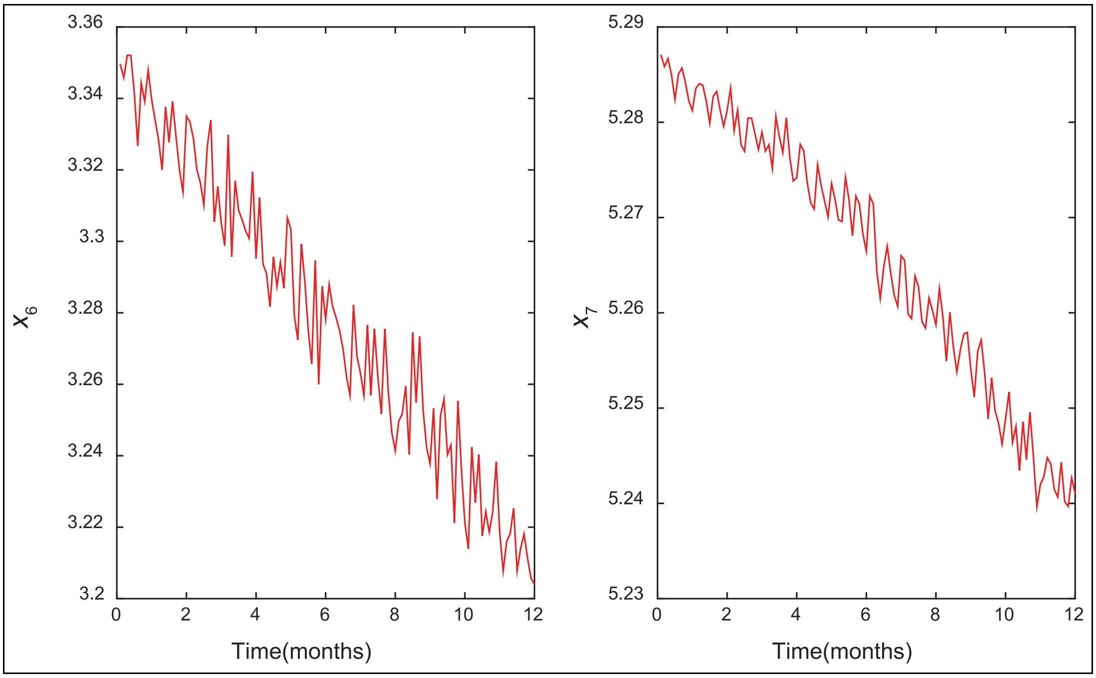


Figure 9. Time-varying data.

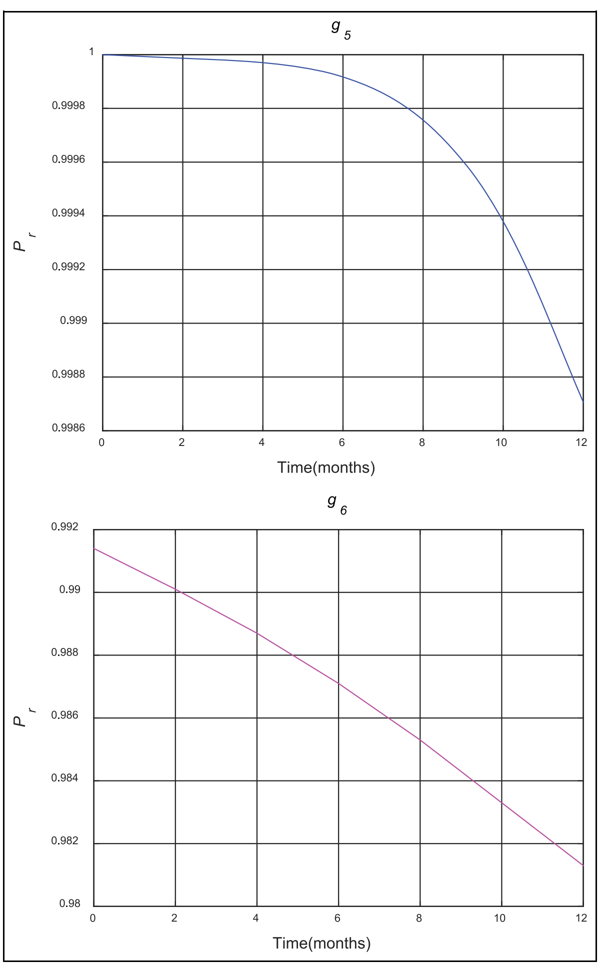


Figure 10. Reliability indexes of g_5 and g_6.

requirements. Although the value of objective function under time-varying uncertainties is 5.23% higher than the results without considering time-varying uncertainties, the reliability performance under time-varying uncertainties has been greatly improved. From this engineering example, it is worth noting that time-varying uncertainties have shown a great influence on the reliability performance of the system. In practical engineering optimization, the impact of time-varying uncertainties should be properly considered.

Conclusion

1. The time-varying uncertainties of complex mechanical system are investigated using stochastic process and corresponding time-varying reliability optimization model are proposed.
2. Using the SAND method, the model of MRDO under time-varying uncertainties is established. Then, a mathematical example and an engineering example are introduced to verify the accuracy and effectiveness of the proposed method.
3. The proposed method is suitable for estimating gradual time-varying uncertainties, such as fatigue strength and material wear, which is not suitable for mutational time-varying uncertainties, such as mutational load and stress. As for mutational time-varying uncertainties, it may be calculated by combining extreme value theory with series reliability analysis method.

4. Due to the complexity of MDO, diverse manifestations, and correlation of time-varying uncertainties, it is very difficult to quantify the characteristics of time-varying uncertainties in some cases, which will be further investigated.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: This research was supported by the National Natural Science Foundation of China under the contract no. 51475082.

References

1. Edward JP, Luis FL and Afzal S. Multidisciplinary design optimization of an automotive magnetorheological brake design. *Comput Struct* 2008; 86: 207–216.
2. Timothy WS and Joaquim R. Multidisciplinary design optimization for complex engineered systems: report from a national science foundation workshop. *J Mech Des: T ASME* 2011; 133: 1–10.
3. Mathieu B, Nicolas B, Philippe D, et al. A survey of multidisciplinary design optimization methods in launch vehicle design. *Struct Multidiscip O* 2012; 45: 619–642.
4. Anand P, Deshmukh, James T, et al. Multidisciplinary dynamic optimization of horizontal axis wind turbine design. *Struct Multidiscip O* 2016; 53: 15–27.
5. Du X and Chen W. Methodology for uncertainty propagation and management in simulation-based systems design. *AIAA J* 2000; 38: 1471–1478.
6. Du X and Chen W. Efficient uncertainty analysis methods for multidisciplinary robust design. *AIAA J* 2002; 40: 545–552.
7. Gu X, Renaud JE, Batill SM, et al. Worst case propagated uncertainty of multidisciplinary systems in robust design optimization. *Struct Multidiscip O* 2000; 20: 190–213.
8. Gao P and Xie L. Fuzzy dynamic reliability models of parallel mechanical systems considering strength degradation path dependence and failure dependence. *Math Probl Eng* 2015; 2015: 1–9.
9. Xie L and Wang Z. Reliability degradation of mechanical components and systems. In: Misra KB (ed.) *Handbook of performability engineering*. London: Springer, 2008, pp.413–429.
10. Zhu SP, Huang HZ, Smith R, et al. Bayesian framework for probabilistic low cycle fatigue life prediction and uncertainty modeling of aircraft turbine disk alloys. *Probabilist Eng Mech* 2013; 34: 114–122.
11. Zhu SP, Huang HZ, Ontiveros V, et al. Probabilistic low cycle fatigue life prediction using an energy-based damage parameter and accounting for model uncertainty. *Int J Damage Mech* 2012; 21: 1128–1153.
12. Son YK. Reliability prediction of engineering systems with competing failure modes due to component degradation. *J Mech Sci Technol* 2011; 25: 1717–1725.
13. Zhang X, Xiao L and Kang J. Degradation prediction model based on a neural network with dynamic windows. *Sensors* 2015; 15: 6996–7015.
14. Zhu SP, Huang HZ, Peng W, et al. Probabilistic Physics of Failure-based framework for fatigue life prediction of aircraft gas turbine discs under uncertainty. *Reliab Eng Syst Safe* 2016; 146: 1–12.
15. Zhu SP, Huang HZ, Li YF, et al. Probabilistic modeling of damage accumulation for time-dependent fatigue reliability analysis of railway axle steels. *Proc IMechE, Part F: J Rail and Rapid Transit* 2015; 229: 23–33.
16. Melchers RE. *Structural reliability analysis and prediction*. Chichester: John Wiley & Sons, 1999.
17. Veneziano D, Grigoriu M and Cornell CA. Vector-process models for system reliability. *J Eng Mech Div: ASCE* 1977; 103: 441–460.
18. Breitung K and Rackwitz R. Nonlinear combination of load processes. *Mech Based Des Struc* 1982; 10: 145–166.
19. Breitung K. Asymptotic crossing rates for stationary Gaussian vector processes. *Stoch Proc Appl* 1988; 29: 195–207.
20. Breitung K. The extreme value distribution of non-stationary vector processes. In: *Proceedings of the 5th international conference on structural safety and reliability (ICOSSAR)*, San Francisco, CA, 7–11 August 1989, pp.1327–1332. AQ4
21. Breitung K. Asymptotic approximations for the crossing rates of Poisson square waves. In: *Proceedings of the conference on extreme value theory and applications*, 1993, pp.82–90. Gaithersburg, MD: NIST Special Publication. AQ5
22. Li CY and Zhang YM. Time-variant reliability assessment and its sensitivity analysis of cutting tool under invariant machining condition based on Gamma process. *Math Probl Eng* 2012; 2012: 542–551.
23. Shinozuka M. Probability of failure under random loading. *J Eng Mech: ASCE* 1964; 90: 147–171.
24. Andrieu-Renaud C, Sudret B and Lemaire M. The PHI2 method: a way to compute time-varying reliability. *Reliab Eng Syst Safe* 2004; 84: 75–86.
25. Cazugu M, Renaud C and Cognard JY. Time-varying reliability of nonlinear structures: application to a representative part of a plate floor. *Qual Reliab Eng Int* 2006; 22: 101–118.
26. Li J and Mourelatos ZP. Time-dependent reliability estimation for dynamic problems using a niching genetic algorithm. *J Mech Design* 2009; 131: 1–13.
27. Jiang C, Ni BY, Han X, et al. Non-probabilistic convex model process: a new method of time-variant uncertainty analysis and its application to structural dynamic reliability problems. *Comput Method Appl M* 2014; 268: 656–676.
28. Wehenkel L, Lebrevelec C, Trotignon M, et al. Probabilistic design of power-system special stability controls. *Control Eng Pract* 1999; 7: 183–194.
29. Yasumasa F, Fabrizio D and Roberto T. Probabilistic design of LPV control systems. *Automatica* 2003; 39: 1323–1337.

30. Wen YK. Reliability-based design under multiple loads. *Struct Saf* 1993; 13: 3–19.
31. Ravindra MK and Walser A. Probabilistic design of nuclear structures: a summary of state of the art and research needs. *Nucl Eng Des* 1978; 50: 115–122.
32. Yan Y and Shi B. Time-dependent reliability analysis under uncertainty. *J Xi'an Jiaotong Univ* 2007; 41: 1303–1306.
33. Allison JT. *Complex system optimization: a review of analytic target cascading collaborative optimization, and other formulations*. PhD Dissertation, University of Michigan, Ann Arbor, MI, 2004.
34. Chan L. *Evaluation of two concurrent design approaches in multidisciplinary design optimization*. NRC report, report no. LM-A-077, January 2001. AQ6
35. Shayanfar M, Abbasnia R and Khodam A. Development of a GA-based method for reliability-based optimization of structures with discrete and continuous design variables using OpenSees and Tcl. *Finite Elem Anal Des* 2014; 90: 61–73.