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Multidisciplinary robust design optimization based on time-varying sensitivity analysis

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Corresponding Author:	Huanwei Xu, Ph.D. University of Electronic Science and Technology of China CHINA
Corresponding Author Secondary Information:	
Corresponding Author's Institution:	University of Electronic Science and Technology of China
Corresponding Author's Secondary Institution:	
First Author:	Huanwei Xu, Ph.D.
First Author Secondary Information:	
Order of Authors:	Huanwei Xu, Ph.D.
	Wei Li
	Mufeng Li
	Cong Hu
	Suichuan Zhang
	Xin Wang
Order of Authors Secondary Information:	
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Suggested Reviewers:	

Multidisciplinary robust design optimization based on time-varying sensitivity analysis

Huanwei Xu[1,*], Wei Li[1], Mufeng Li[1], Cong Hu[1], Suichuan Zhang[1], Xin Wang[1]

[1]School of Mechatronics Engineering, University of Electronic Science and Technology of China, Chengdu, Sichuan, 611731, PR China

Abstract: The performance of a complex mechanical system often degrades over time, which is mainly caused by time-varying uncertainties. How to deal with time-varying uncertainties in multidisciplinary design optimization (MDO) is a key factor to improve the design of complex mechanical systems. Considering time-varying uncertainties in mechanical systems, a multidisciplinary robust design optimization (MRDO) method is put forward based on time-varying sensitivity analysis. Firstly, time-varying reliability indexes of limit state functions by combining sensitivity analysis and empirical correction formula is calculated; Then, the propagation effects of these time-varying uncertainties are qualified through combining the simplified Implicit Uncertainty Propagation (IUP) method and Sequential Quadratic Programming (SQP) method, then robust design method is integrated into MDO method to reduce the impact of time-varying uncertainties; Finally, the illustration of the proposed method is provided with both of a mathematical problem and an engineering example.

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1. Introduction

The design optimization of a mechanical system is difficult, which generally includes highly nonlinear objective functions and constraints. Furthermore, multi-sources of uncertainties widely exist in complex mechanical systems, such as load fluctuations; material properties; geometry sizes; operation modes and so on. The design optimization results are often inaccurate while ignoring these uncertainties and their propagation effects. Many researchers have done a lot of work on multidisciplinary design optimization (MDO) under time-invariant uncertainties [1-10].

However, many facts show that the degradation failure is one of the main reasons for complex mechanical systems losing their functions [11-14]. The degradation failure is mainly caused by time-varying uncertainties, such as strength and performance degradation, material aging, wear, oxidation, corrosion and so on. Until now, many research methods had been developed to deal with time-varying uncertainties [15-19]. Savage and Son [20] analyzed product reliability in life cycle on condition that performance degradation, random process force, random process parameters are monotonous. Singh and Mourelatos [21] calculated time-varying reliability of non-monotonic unrepairable system with series reliability method by considering several key time intervals in life cycle. Royset [22] applied time-varying reliability optimization design method to maximize the product value in the whole product life cycle. Li and Mourelatos [23] combined Niche Genetic Algorithm and MPP based reliability method to solve dynamic reliability problem. Renaud and Sudret [24] proposed PHI2 method to solve time-varying reliability problem for the first time, and then deduced more accurate model to calculate time-varying reliability based on PHI2 method [25]. Kuschel and Rackwits [26] used the crossing rate method for the first time to optimize time-varying structure. Wang and Wang [27] proposed nested extreme response surface method to analyze time-varying reliability problem. From above-mentioned achievements, note that the theories and methods to solve time-varying reliability problem in single discipline are rather mature. However, the modern mechanical systems often consist of multiple disciplines, where the impact of time-vary uncertainties will be transferred from one discipline to another discipline. Furthermore, the time-varying uncertainties are diversely manifested and often correlated with each other which makes it very difficult to evaluate the impact of time-varying uncertainties.

Various uncertainties lead to performance fluctuation of a mechanical system. To eliminate these uncertainties is difficult and costly. To reduce the effects of these uncertainties is relatively easy and economical. Robust design is such a method to keep the performance robust under various uncertainties [4-5, 7, 28-36] .

Aiming at dealing with time-varying uncertainties in MDO, this paper tends to propose a multidisciplinary robust design optimization (MRDO) method under time-varying uncertainties. The remainder of this paper is arranged as follows. In Section 2, the characteristics of time-varying uncertainties is analyzed, and then empirical correction formula is applied to calculate the sensitivity index of time-varying reliability; In Section 3, a model to qualify the propagation of time-varying uncertainties is

[*] Corresponding author.
E-mail address: hwxu@uestc.edu.cn

established by using Implicit Uncertainty Propagation (IUP) method; In Section 4, a MRDO framework under time-varying uncertainties is proposed through combining robust design method; In Section 5, a mathematical problem and an engineering example are introduced to illustrate the feasibility and effectiveness of the proposed method. The last section gives the research conclusion.

2. Time-varying reliability sensitivity analysis

Until now, there have been many achievements in reliability sensitivity analysis in the design and optimization of mechanical systems [37-42]. If one source of uncertainty has a great effect on mechanical system performance (high reliability sensitivity), we should strictly control this uncertainty in the process of design and manufacture to ensure that the product has enough reliability in the whole life cycle. Otherwise, if an uncertainty has less or no effect on mechanical system performance (low reliability sensitivity), this uncertainty will be treated as deterministic factor to simplify the design process and improve computational efficiency. The system reliability can be viewed as a dynamic time-varying process due to time-varying uncertainties, such as material properties, operating environments and loads. Thus, the effect of time, which is called time-varying reliability sensitivity analysis, should be considered.

Assume a random variable vector is $X=[x_1, x_2, \ldots, x_n]^T$, the index of reliability sensitivity can be calculated by following forma:

$$\frac{dR(t)}{d(\bar{X},t)^T} = \frac{\partial R(\beta(t))}{\partial \beta(t)} \frac{\partial \beta(t)}{\partial \mu_{g(t)}} \frac{\partial \mu_{g(t)}}{\partial (\bar{X},t)^T} \tag{1}$$

where

$$\frac{\partial R(\beta(t))}{\partial \beta(t)} = \varphi(\beta(t)); \quad \frac{\partial \beta(t)}{\partial \mu_{g(t)}} = \frac{1}{\sigma_{g(t)}};$$

$$\frac{\partial \mu_{g(t)}}{\partial X^T} = \begin{bmatrix} \frac{\partial g(t)}{\partial x_1} & \frac{\partial g(t)}{\partial x_2} & \cdots & \frac{\partial g(t)}{\partial x_n} \end{bmatrix}$$

However, Eq. (1) is not suitable for solving the limit state function with high nonlinearity. When the design variables are normal distributed, the following modified formula can be derived as

$$\frac{dR(t)}{d(\bar{X},t)^T} = \frac{\partial R(\beta(t))}{\partial \beta(t)} \frac{\partial \beta(t)}{\partial \mu_{g(t)}} \frac{\partial \mu_{g(t)}}{\partial (\bar{X},t)^T} + \frac{\partial R(\beta(t))}{\partial \beta(t)} \frac{\partial \beta(t)}{\partial \sigma_{g(t)}} \frac{\partial \sigma_{g(t)}}{\partial (\bar{X},t)^T} \tag{2}$$

where

$$\frac{\partial \beta(t)}{\partial \sigma_{g(t)}} = -\frac{\mu_{g(t)}}{\sigma_{g(t)}^2}$$

$$\frac{\partial \sigma_{g(t)}}{\partial X^T} = \frac{1}{2\sigma_{g(t)}} \begin{bmatrix} \frac{\partial^2 g(t)}{\partial (X^T)^2} \otimes \frac{\partial g(t)}{\partial X^T} + \\ \left(\frac{\partial^2 g(t)}{\partial (X^T)^2} \otimes \frac{\partial g(t)}{\partial X^T} \right)(I \otimes U) \end{bmatrix} (I \otimes Var(X))$$

According to Edgeworth series method [43] and Eq. (2), the index of time-varying reliability sensitivity of random variable vector X with a certain distribution can be calculated as:

$$\begin{aligned} \frac{dR(t)}{d(\bar{X},t)^T} &= \frac{\partial R(\beta(t))}{\partial \beta(t)} \frac{\partial \beta(t)}{\partial u_{g(t)}} \frac{\partial \mu_{g(t)}}{\partial (\bar{X},t)^T} + \\ &\left[\frac{\partial R(\beta(t))}{\partial \beta(t)} \frac{\partial \beta(t)}{\partial \sigma_{g(t)}} + \frac{\partial R(\beta(t))}{\partial \sigma_{g(t)}} \right] \frac{\partial \sigma_{g(t)}}{\partial (\bar{X},t)^T} \end{aligned} \tag{3}$$

where

$$\frac{\partial R(\beta(t))}{\partial \beta(t)} = \varphi(-\beta(t)) \left\{ \begin{aligned} &\left[1-\beta(t) \begin{bmatrix} \frac{1}{6}\frac{\theta_{g(t)}}{\sigma_{g(t)}^3} H_2(-\beta(t)) + \\ \frac{1}{24}\left(\frac{\eta_{g(t)}}{\sigma_{g(t)}^4} - 3 \right) H_3(-\beta(t)) + \\ \frac{1}{72}\left(\frac{\theta_{g(t)}}{\sigma_{g(t)}^3} \right)^2 H_5(-\beta(t)) \end{bmatrix} \right] \\ &- \begin{bmatrix} \frac{1}{3}\frac{\theta_{g(t)}}{\sigma_{g(t)}^3} H_1(-\beta(t)) + \\ \frac{1}{8}\left(\frac{\eta_{g(t)}}{\sigma_{g(t)}^4} - 3 \right) H_2(-\beta(t)) + \\ \frac{5}{72}\left(\frac{\theta_{g(t)}}{\sigma_{g(t)}^3} \right)^2 H_4(-\beta(t)) \end{bmatrix} \end{aligned} \right\} \tag{4}$$

$$\frac{\partial R(\beta(t))}{\partial \sigma_{g(t)}} = \varphi(-\beta(t)) \begin{bmatrix} \frac{1}{2}\frac{\theta_{g(t)}}{\sigma_{g(t)}^4} H_2(-\beta(t)) + \\ \frac{1}{6}\frac{\eta_{g(t)}}{\sigma_{g(t)}^5} H_3(-\beta(t)) + \\ \frac{1}{12}\frac{\theta_{g(t)}^2}{\sigma_{g(t)}^7} H_5(-\beta(t)) \end{bmatrix} \tag{5}$$

$$\begin{aligned} &\mu_{g(t)} = \bar{g}(t); \sigma_{g(t)}^2 = \left(\frac{\partial g(t)}{\partial X^T} \right)^{[2]} V^2(X); \\ &\theta_{g(t)} = \left(\frac{\partial g(t)}{\partial X^T} \right)^{[3]} V^3(X); \eta_{g(t)} = \left(\frac{\partial g(t)}{\partial X^T} \right)^{[4]} V^4(X) \end{aligned} \tag{6}$$

In Eq. (6), $(\bullet)^{[k]} = (\bullet)\otimes(\bullet)\otimes\cdots\otimes(\bullet)$ is Kronecker product, V^k represents the k-th moment operator. $H_j(y)$ is j-th Hermite polynomial, the recurrence relations are as follows:

$$\begin{cases} H_{j+1}(y) = yH_j(y) - jH_{j-1}(y) \\ H_0(y) = 1, \qquad H_1(y) = y \end{cases}$$

If Edgeworth series method is used to calculate reliability, it sometimes may appear the reliability greater than 1. In this case, the empirical correction formula is more accurate in reliability analysis than the Edgeworth series method [40]. The empirical correction formula is given as follows:

$$R^*(\beta(t)) = R(\beta(t)) - \frac{R(\beta)-\Phi(\beta(t))}{\left\{1+\left[R(\beta(t))-\Phi(\beta(t))\right]\beta(t)\right\}^{\beta(t)}} \tag{7}$$

where the index of reliability sensitivity $\beta(t)$ can be deduced from the differential of empirical formula:

$$\begin{aligned}\frac{\partial R^*(t)}{\partial\beta(t)} &= \frac{\partial R(\beta)}{\partial\beta(t)} + \left[\frac{\partial R(\beta)}{\partial\beta(t)} - \varphi(\beta(t))\right]\times \\ &\frac{\beta(t)(\beta(t)-1)\left[R(\beta)-\Phi(\beta(t))\right]-1}{\left\{1+\left[R(\beta)-\Phi(\beta(t))\right]\beta(t)\right\}^{\beta(t)+1}} \\ &+\frac{\left[R(\beta)-\Phi(\beta(t))\right]\left\{1+\left[R(\beta)-\Phi(\beta(t))\right]\beta(t)\right\}\ln\left\{1+\left[R(\beta)-\Phi(\beta(t))\right]\beta(t)\right\}}{\left\{1+\left[R(\beta)-\Phi(\beta(t))\right]\beta(t)\right\}^{\beta(t)+1}} \\ &+\frac{\left[R(\beta)-\Phi(\beta(t))\right]^2\beta(t)}{\left\{1+\left[R(\beta)-\Phi(\beta(t))\right]\beta(t)\right\}^{\beta(t)+1}}\end{aligned} \tag{8}$$

Through replacing $\frac{\partial R(\beta)}{\partial\beta(t)}$ in Eq. (3) with Eq. (5), the index of time-varying reliability sensitivity can be calculated.

3. Simplified robust optimization design based on IUP method

The complex mechanical system consists of different sub-systems. Each subsystem has specific function. The coupled relationships between subsystems exist, which make it very difficult to evaluate the effect of time-varying uncertainties. The time-varying uncertainties in one subsystem not only have effect on its own subsystem, but also have effect on another subsystem because of coupled relationship between these two subsystems. In this paper, aiming at dealing with time-varying uncertainties, simplified robust optimization design based on IUP method is proposed to avoid complicated global sensitivity equation (GSE) and improve computational efficiency.

3.1 Uncertainty analysis of MDO

Generally, the propagation of uncertainties between subsystems cannot be calculated by simple decomposed and superposed method. How to evaluate iterative effect of coupled variables and system performance under uncertainties are key factors in MDO.

The propagation of uncertainties is shown in Fig. 1. Note that each coupled state variable is no only an input variable of one subsystem, but also an output variable of another subsystem. For example, the coupled state variable y_{12} is the output variable of subsystem 1 and also the input variable of subsystem 2, which makes uncertainties have coupled iterative effect.

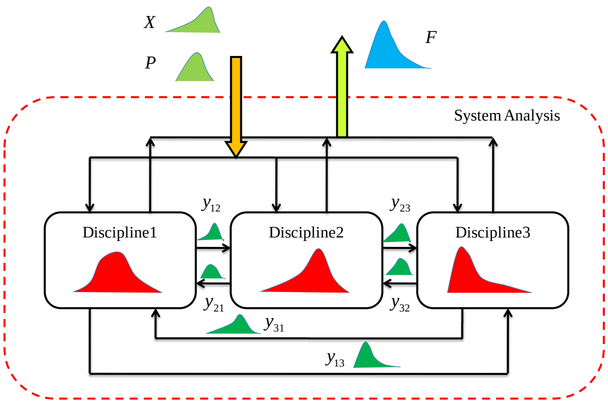


Fig. 1 The propagation of uncertainties in MDO

3.2 Uncertainty Modeling of MDO

For the uncertainty model of MDO, the key issue is to calculate uncertainties of coupled state variables. In reference [5], the maximal deviation of coupled state variables can be expressed by following formula:

$$\begin{Bmatrix}\Delta y_1\\ \Delta y_2\\ \Delta y_3\end{Bmatrix} = \begin{Bmatrix}\left|\frac{dy_1}{dX}\right|\\ \left|\frac{dy_2}{dX}\right|\\ \left|\frac{dy_3}{dX}\right|\end{Bmatrix}|\Delta X| + \begin{pmatrix}|B_{11}| & |B_{12}| & |B_{13}|\\ |B_{21}| & |B_{22}| & |B_{23}|\\ |B_{31}| & |B_{32}| & |B_{33}|\end{pmatrix}\begin{Bmatrix}\delta_1(X,y_2,y_3)\\ \delta_2(X,y_1,y_3)\\ \delta_3(X,y_1,y_2)\end{Bmatrix} \tag{9}$$

where

$$\begin{pmatrix}|B_{11}| & |B_{12}| & |B_{13}|\\ |B_{21}| & |B_{22}| & |B_{23}|\\ |B_{31}| & |B_{32}| & |B_{33}|\end{pmatrix} = \begin{pmatrix}I_1 & -\frac{\partial T_1}{\partial y_2} & -\frac{\partial T_1}{\partial y_2}\\ -\frac{\partial T_2}{\partial y_1} & I_2 & -\frac{\partial T_2}{\partial y_3}\\ -\frac{\partial T_3}{\partial y_1} & -\frac{\partial T_3}{\partial y_2} & I_3\end{pmatrix}^{-1}$$

I_1, I_2 and I_3 are unit matrixes; T_1, T_2 and T_3 represent the models of each subsystem analysis, respectively; δ_1, δ_2 and δ_3 are model errors of each subsystem, the total derivative can be obtained by GSE:

$$\begin{Bmatrix} \frac{dy_1}{dX} \\ \frac{dy_2}{dX} \\ \frac{dy_3}{dX} \end{Bmatrix} = \begin{pmatrix} I_1 & -\frac{\partial T_1}{\partial y_2} & -\frac{\partial T_1}{\partial y_2} \\ -\frac{\partial T_2}{\partial y_1} & I_2 & -\frac{\partial T_2}{\partial y_3} \\ -\frac{\partial T_3}{\partial y_1} & -\frac{\partial T_3}{\partial y_2} & I_3 \end{pmatrix}^{-1} \begin{Bmatrix} \frac{\partial T_1}{\partial X} \\ \frac{\partial T_2}{\partial X} \\ \frac{\partial T_3}{\partial X} \end{Bmatrix} \tag{10}$$

Since GSE method is a time-consuming method for sensitivity analysis, a simplified IUP method [42] can also be used to solve this issue. According to simplified IUP method, we take the deviations of coupled state variables as auxiliary design variables instead of calculating GSE and local partial derivative. Considering design variables errors and analysis errors, the model is given as follows:

$$\begin{aligned}
&\min \quad F = \omega_1 \frac{f}{f^*} + \omega_2 \frac{\Delta f}{\Delta f^*} \\
&\quad \Delta f(X_d, X, Y, P(t)) = \sum_{i=1}^{s_{design}} \left| \frac{\partial f(X_d, X, Y, P(t))}{\partial X_i} \Delta X_i \right| + \\
&\quad\quad \sum_{j=1}^{s_{auxiliary}} \left| \frac{\partial f(X_d, X, Y, P(t))}{\partial Y_j} \Delta Y_j \right| \\
&s.t. \quad g_i(X_d, X, Y, P(t)) + \Delta g_i(X_d, X, Y, P(t)) \le 0 \\
&\quad \Delta g_i(X) = \sum_{j=1}^{s_{design}} \left| \frac{\partial g_i(X_d, X, Y, P(t))}{\partial X_j} \Delta X_j \right| + \\
&\quad\quad \sum_{j=1}^{s_{auxiliary}} \left| \frac{\partial g_i(X_d, X, Y, P(t))}{\partial Y_j} \Delta Y_j \right| \\
&\quad \Delta Y = \sum_{j=1}^{s_{design}} \left| \frac{\partial T(X_d, X, Y, P(t))}{\partial X_j} \Delta X_j \right| + \\
&\quad\quad \sum_{j=1}^{s_{auxiliary}} \left| \frac{\partial T(X_d, X, Y, P(t))}{\partial Y_j} \Delta Y_j \right| + \Delta \varepsilon \\
&\quad X_R^L = X^L + \Delta X;\ X_R^U = X^U - \Delta X; \\
&\quad (X_d)_R^L = (X_d)^L + \Delta(X_d); \\
&\quad (X_d)_R^U = (X_d)^U - \Delta(X_d); \\
&\quad Y_R^L = Y^L + \Delta Y;\ Y_R^U = Y^U - \Delta Y
\end{aligned} \tag{11}$$

where X_d are deterministic design variables, X are time-varying design variables, Y are coupled state variables, $P(t)$ are time-varying design parameters, ΔX_i means deviation of the i-th design variable, ΔY_j represents deviation of the j-th auxiliary design variable, $\Delta f(\cdot)$ is deviation of $f(\cdot)$, $T(\cdot)$ is discipline analysis model, $s_{desiagn}$ is the number of design variables, $s_{auxiliary}$ is the number of coupled state variables. ω_1, ω_2 are weight factors, f^* and Δf^* are values of objective function F when $[\omega_1, \omega_2]=[0,1]$ and $[\omega_1, \omega_2]=[1,0]$, respectively. $\Delta\varepsilon$ is error of analysis model, $(\cdot)^U$ and $(\cdot)^L$ are upper and lower limits of design vector, respectively. $(\cdot)_R^U$ and $(\cdot)_R^L$ are upper and lower limits of robust design vector, respectively.

4. RMDO based on time-varying sensitivity analysis

MDO is an optimal design methodology for complicated mechanical system. Its complexity is mainly reflected in computational complexity, organizational complexity, the complexities of the model structure and information exchange. The mathematical complexity of time-varying uncertainties further exacerbates the difficulty in modeling and solving the MDO problem.

The reliability design method and robust design method are effective to improve product performance under time-varying uncertainties. However, these two methods are usually applied in different design stages independently. In this section, time-varying sensitivity analysis and MRDO method are combined to reduce computational and system analysis costs. The formula of the proposed method is as follows:

$$\begin{aligned}
&\begin{cases}
f = \min f(X_d, X, Y, P(t)) \\
\Delta f(X_d, X, Y, P(t)) = \sum_{i=1}^{s_{design}} \left| \frac{\partial f(X_d, X, Y, P(t))}{\partial X_i} \Delta X_i \right| + \\
\quad \sum_{j=1}^{s_{auxiliary}} \left| \frac{\partial f(X_d, X, Y, P(t))}{\partial Y_j} \Delta Y_j \right| \\
f_R = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial R(t)}{\partial X_i} \right)^2} \\
F = \omega_1 \frac{f}{f^*} + \omega_2 \frac{\Delta f}{\Delta f^*} + \omega_3 \frac{f_R}{\Delta f_R^*}
\end{cases} \\
&s.t. \quad R(t) - [R] \ge 0 \\
&\quad g_i(X_d, X, Y, P(t)) + \Delta g_i(X_d, X, Y, P(t)) \le 0 \\
&\quad Y = T_{displine}(X_d, X, Y, P(t)) \\
&\quad \Delta Y = \sum_{i=1}^{s_{design}} \left| \frac{\partial T(X_d, X, Y, P(t))}{\partial X_i} \Delta X_i \right| + \\
&\quad\quad \sum_{j=1}^{s_{auxiliary}} \left| \frac{\partial T(X_d, X, Y, P(t))}{\partial Y_j} \Delta Y_j \right| \\
&\quad X^{L_R} = X^L + \Delta X;\ X^{U_R} = X^U - \Delta X; \\
&\quad (X_d)_R^L = (X_d)^L + \Delta(X_d); \\
&\quad (X_d)_R^U = (X_d)^U - \Delta(X_d); \\
&\quad Y^{L_R} = Y^L + \Delta Y;\ Y^{U_R} = Y^U - \Delta Y
\end{aligned} \tag{12}$$

where $f_R(\cdot)$ is the objective function of time-varying sensitivity analysis; $T_{displine}(\cdot)$ is discipline analysis model; [R] is a designed reliability target.

The framework of MRDO under time-varying uncertainties is shown in Fig. 2. The reliability sensitivity of each time-varying design variable is calculated in the module of time-varying reliability sensitivity; the simplified IUP method in the module of MRDO is introduced to guarantee the reliability and robustness of optimization result. The MDO framework will lead to an optimal result.

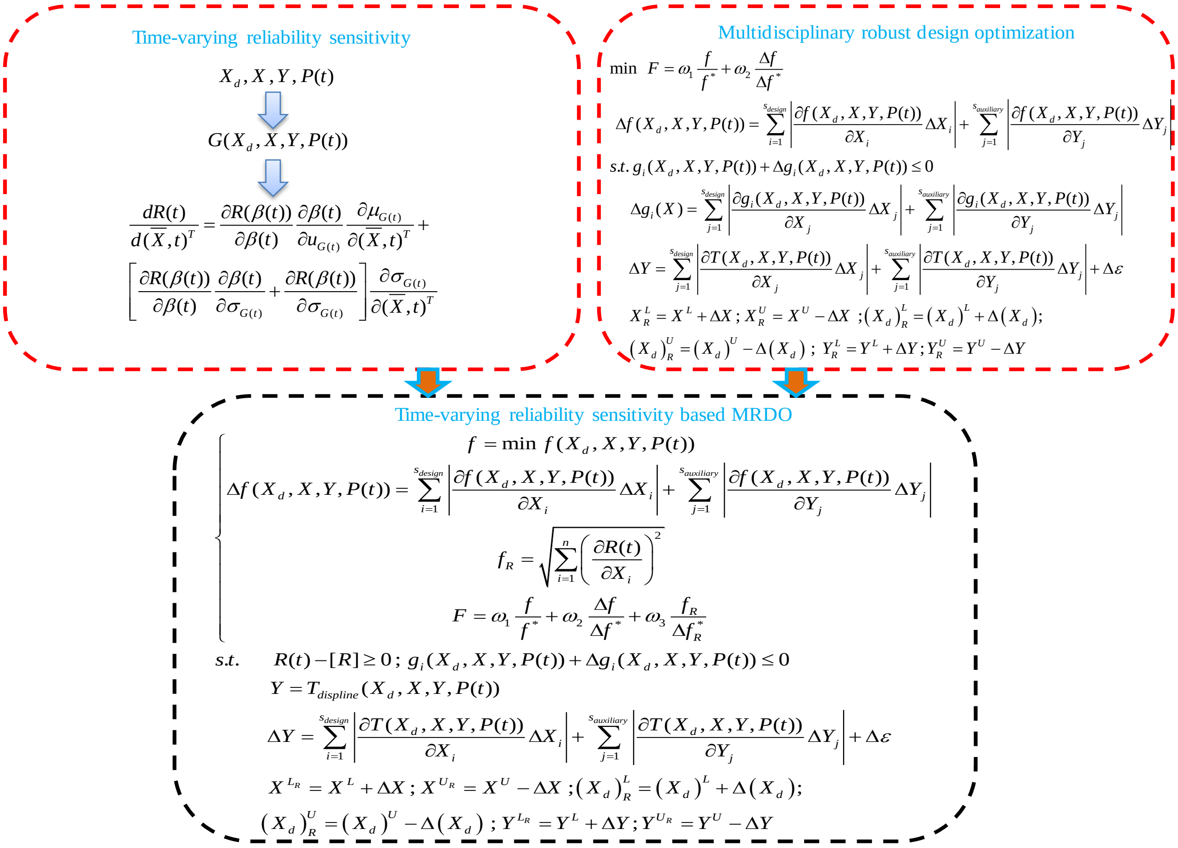


Fig. 2 The framework of MRDO under time-varying uncertainties

5. Case Studies

In this section, both a mathematical example and a case study of an engineering system are introduced to illustrate the feasibility and validity of the proposed method.

5.1 A mathematical example

A classic MDO problem including two disciplines is used for model verification [45]. Each discipline has a coupled state variable, nonlinear coupled relationship exists between two disciplines.

The optimization model is expressed as

$$Min \qquad f = x_1^2 + x_2 + y_1 + e^{-y_2}$$

$$\begin{aligned} s.t. \qquad & g_1 = 1 - \frac{y_1}{3.16} \le 0 \\ & g_2 = \frac{y_2}{24} - 1 \le 0 \\ & h_1 = x_1 + x_2 + x_3^2 - 0.2y_2 - y_1 \\ & h_2 = \sqrt{y_1} + x_2 + x_3 - y_2 \\ & 1 \le x_1 \le 10 \\ & 1 \le x_2 \le 10 \\ & -10 \le x_3 \le 10 \end{aligned}$$

where

$$y_1 = x_1 + x_2 + x_3^2 - 0.2y_2$$
$$y_2 = \sqrt{y_1} + x_2 + x_3$$

The corresponding MDO model is shown in Fig. 3.

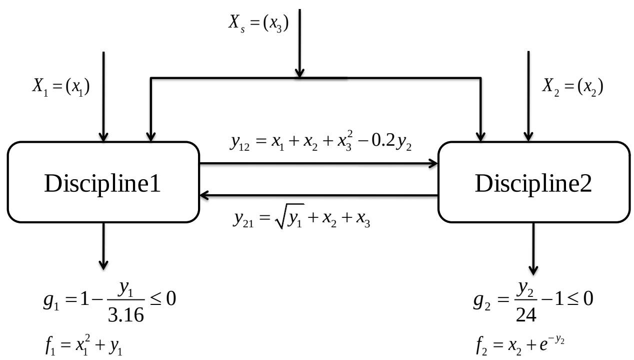


Fig.3 The MDO model of mathematical example

The corresponding limit state function is given by $G(t,\omega) = 2.6 - 1.6x_3^2 + P(t)x_1x_2$. The data of $P(t)$ during 1000 days are obtained by MATLAB simulation, which is shown in Fig. 4. Assuming that x_1, x_2, x_3 are time-varying uncertainties and corresponding degradation trends, respectively

$$x_1 = x_1 e^{-0.0002*t},\ x_2 = x_2 \pm \varepsilon,\ x_3 = x_3 e^{-0.0001*t}$$

where ε is a small real number. Set ΔX=0.01X during MRDO optimization process. x_1, x_2, x_3 are lognormal distributed and corresponding standard deviations are 0.1, 0.1, 0.14.

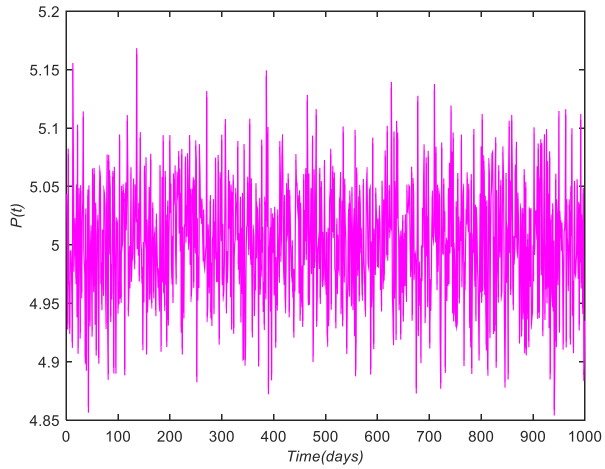


Fig. 4 Time-varying data of $P(t)$

The model of MRDO based on time-varying sensitivity analysis is established according to the framework mentioned in section 4. A comparison on the optimization results of conventional MDO method and the proposed method are listed in Table 1. The corresponding iteration processes of objective function and constraints are shown in Fig. 5-Fig. 6, respectively. The time-varying sensitivity indexes of these two methods are shown in Fig. 7.

The optimization variation is obtained by following formula:

$$\text{Optimization variation} = \frac{f_{Novel} - f_{Original}}{f_{Original}} \times 100\% \tag{13}$$

Table 1 Optimization results

	x_1	x_2	x_3	y_1	y_2	f
Original design	1.0000	1.0000	1.4136	3.1600	4.1912	5.1751
Novel design	1.0000	1.5844	1.2529	3.2274	4.6338	5.8215

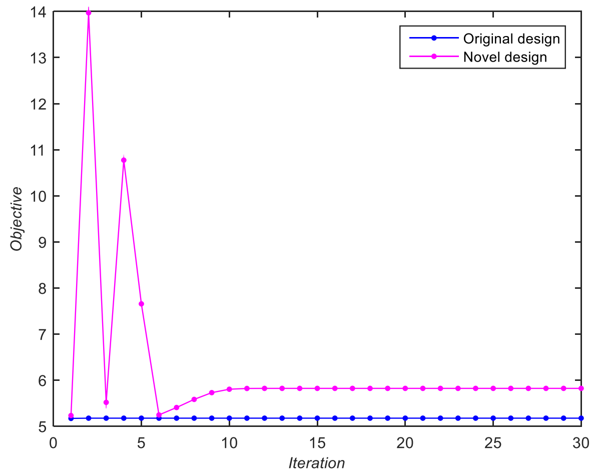


Fig. 5 The iteration process of objective function

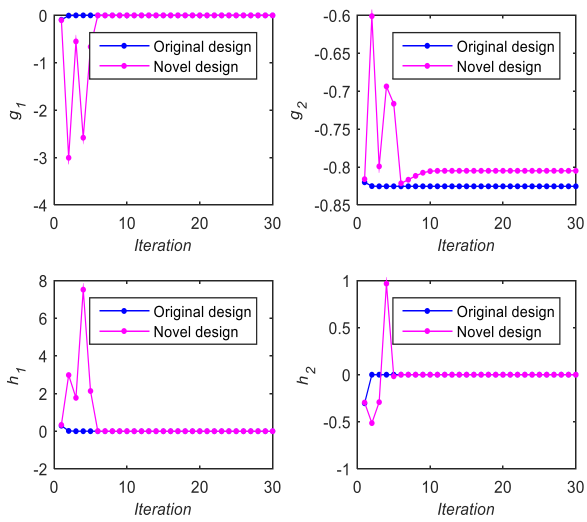


Fig. 6 The iteration processes of constraints

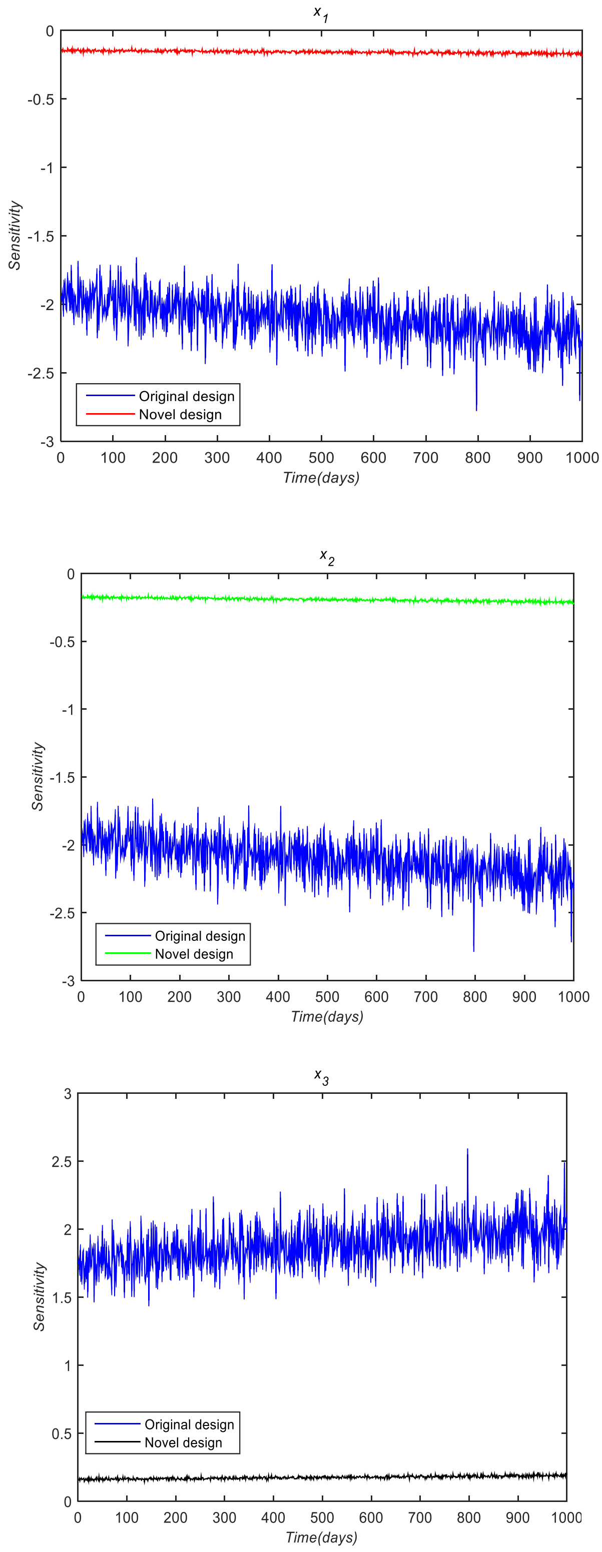
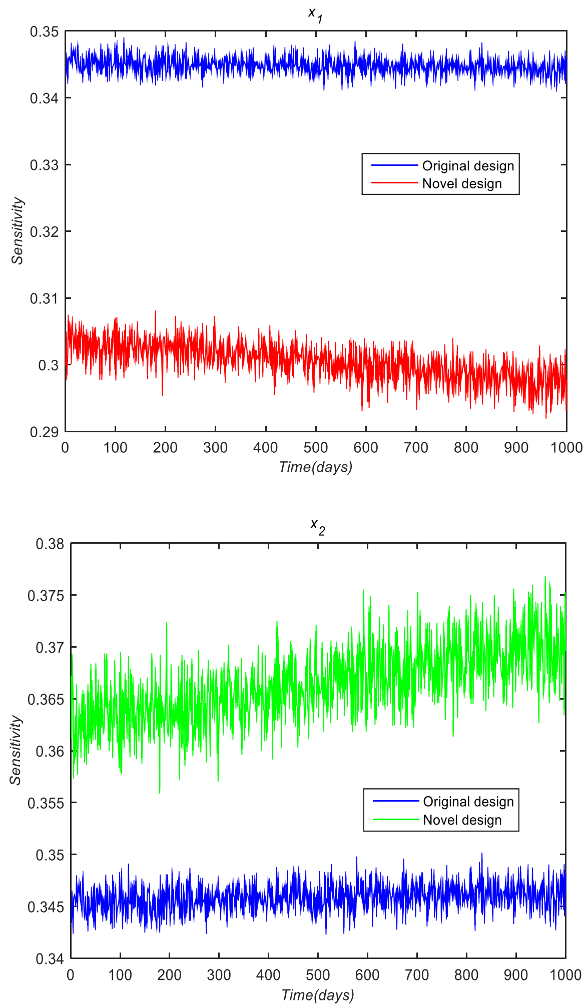


Fig. 7 The comparison of time-varying sensitivity indexes

The sensitivity proportions of time-varying uncertainties can be calculated by

$$RS(t)_i^{proportion} = \frac{|R_i(t)|}{\sum_{i=1}^{m} |R_i(t)|} \quad (14)$$

The corresponding results of sensitivity proportions are shown in Fig. 8.



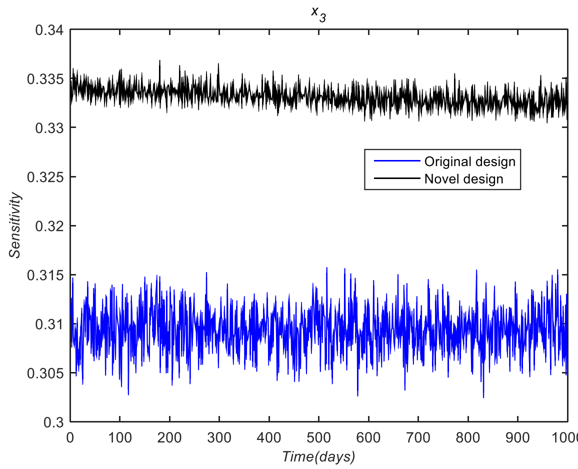
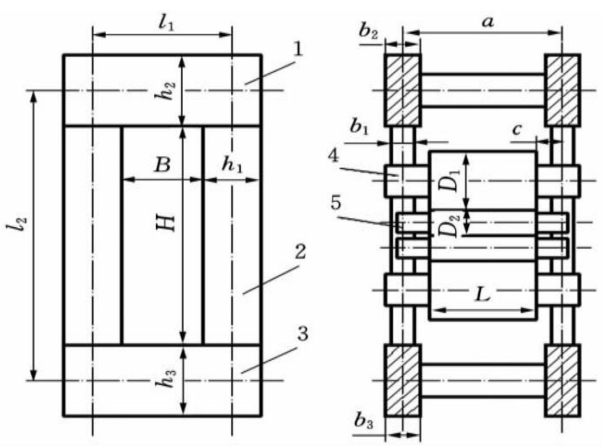


Fig. 8 Comparison of sensitivity proportions

From the optimization results in Fig. 6, note that both of the constraints of these two methods meet the design requirements. Although the optimization result of the proposed method is increased by 12.49% than that of conventional design, it can be seen from Fig. 7 that the time-varying sensitivity index of conventional method increases over time and has large fluctuation, which means the time-varying uncertainties x_1, x_2, x_3 have shown great influences on performance function. On the other hand, the time-varying sensitivity index of the proposed method are rather smaller and shows smaller fluctuation, which means the time-varying uncertainties x_1, x_2, x_3 have shown smaller influence on performance function. A conclusion from Fig. 8 can be draw that the sensitivity proportion of the proposed method prefers to x_2 which is insensitive over time and that means our optimization result is robust.

5.2 An engineering example

In this section, a design optimization problem of the four-high rolling mill stand is introduced [46]. The simplified structure diagram of roller base is shown in Fig. 10.



1. Upper beam. 2. Column 3. Lower beam 4. Supporting roll 5. Working roll

Fig. 10. Equivalent simplified structure diagram of four-high rolling mill stand

There are seven desing variables $x=(x_1,x_2,x_3,x_4,x_5,x_6,x_7)^T=(h_1,b_1,h_2,b_2,h_3,b_3,D_1)^T$.

x_1 is the height of column cross-section h_1, x_2 presents the width of column cross section b_1, x_3 is the average height of upper beam cross section h_2, x_4 presents the width of upper beam cross section b_2, x_5 is the height of lower beam cross section h_3, x_6 presents the width of lower beam cross section b_3, x_7 is the diameter of supporting roll D_1.

The objective function is composed of six parts:

(1) The bending deformations of lower and upper beams which is caused by the bending moment.

$$f_1(x)=1.91\times10^{-6}\times(x_1+0.59)^3\times\left(1/\left(x_4x_3^3\right)+1/\left(x_6x_5^3\right)\right)\times \left\{1-0.75\times\left[\begin{array}{l}1+(x_3+x_5+4.3)x_4x_2^3x_6x_5^3\,/\\ \left((x_1+0.59)x_2x_1^3\left(x_4x_3^3+x_6x_5^3\right)\right)\end{array}\right]^{-1}\right\}$$

(2) The bending deformations of the upper and lower beams which is caused by shear stress.

$$f_2(x)=3.704\times10^{-6}\times(x_1+0.59)\times\left(1/(x_3x_4)+1/(x_5x_6)\right)$$

(3) Tensile deformation of the columns.

$$f_3(x)=5.119\times10^{-6}\times1/\,x_1x_2$$

(4) The bending deformations of supporting rollers which is caused by the bending moment.

$$f_4(x)=0.9671\times10^{-6}\times x_7^{-4}\times \left[\begin{array}{l}8(x_2+0.656)^3-0.64(x_2+0.656)+\\ 0.64+8(x_3+0.256)^3\times\left(218.8x_7^4-1\right)\end{array}\right]$$

(5) The sum of the bending deformation which is caused by shear stress.

$$f_5(x)=1.533\times10^{-5}\times x_7^{-2}\times\left[\begin{array}{l}(x_2+0.656)-0.2+\\ (x_2+0.256)\times\left(14.79x_7^2-1\right)\end{array}\right]$$

(6) The sum of elastic squash deformations between working rolls and supporting rolls.

$$f_6(x)=0.263\times10^{-4}\ln\left[0.5904\times10^5\times(x_7+0.28)\right]$$

There are twelve constrains in the optimization model of four-high rolling mill stand:

(1) The dimension restrictions g_1, g_2, the contact strength of rolls g_3, the bending strength of dangerous section of supporting roll g_4.

$$g_1(x)=x_6-0.42\le 0\,;\; g_2(x)=0.336-x_7\le 0;$$
$$g_3(x)=0.89\times10^6\times\sqrt{1+0.28/x_7}-1.61\times10^6\le 0;$$
$$g_4(x)=0.1678\times10^6\times(x_2+0.256)-0.125\times10^6\le 0$$

(2) The composite tensile and bending strength of column

g_5, $[\sigma_P]=0.055\times10^6 KN/m^2$

$$g_5(x)=\frac{500}{x_1x_2}+\frac{750(x_1+0.59)}{x_1^2x_2}\times\left[1\Big/\left(1+\left((x_3+x_5+4.3)x_4x_3^3x_6x_5^3\right)\Big/\left(\begin{matrix}(x_1+0.59)x_2x_1^3\\(x_4x_3^3+x_6x_5^3)\end{matrix}\right)\right)\right]-[\sigma_P]\le 0$$

(3) The bending strength of upper and lower beams.

$$g_6(x)=1.5\times10^3(x_1+0.59)x_4^{-1}x_3^{-2}\times\left\{1-0.5\Big/\left[1+(x_3+x_5+4.3)x_4x_3^3x_6x_5^3\Big/\left(\begin{matrix}(x_1+0.59)x_2x_1^3\\(x_4x_3^3+x_6x_5^3)\end{matrix}\right)\right]\right\}-0.055\times10^6\le 0$$

$$g_7(x)=1.5\times10^3(x_1+0.59)x_6^{-1}x_5^{2}\times\left\{1-0.5\Big/\left[1+(x_3+x_5+4.3)x_4x_3^3x_6x_5^3\Big/\left(\begin{matrix}(x_1+0.59)x_2x_1^3\\(x_4x_3^3+x_6x_5^3)\end{matrix}\right)\right]\right\}-0.055\times10^6\le 0$$

(4) Height and width constraints.

$$g_8(x)=x_2-x_1\le 0\,;\; g_9(x)=x_4-x_3\le 0\,;$$
$$g_{10}(x)=x_6-x_5\le 0\,; g_{11}(x)=0.26-x_2\le 0\,;$$
$$g_{12}(x)=x_3-2.5x_4\le 0\,;\; g_{13}(x)=x_5-2.5x_6\le 0;$$

(5) The weight of structure is not more than the existing similar four-high rolling mill stand's.

$$g_{14}(x)=15.6\times\left[2.15x_1x_2+(x_1+0.295)(x_3x_4+x_5x_6)\right]-7.484\le 0;$$

According to the MDO concept, four-high rolling mill stand can be divided into three subsystems:1) beams; 2) columns; 3) supporting roll. The MDO framework is shown in Fig. 11.

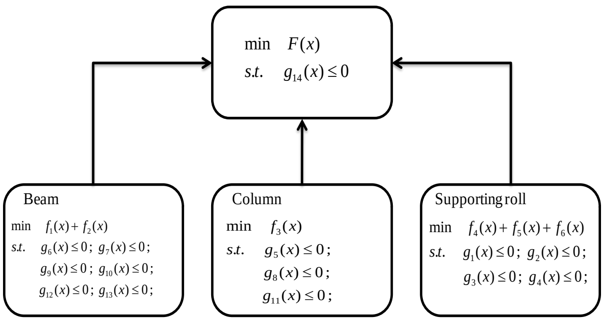


Fig. 11 MDO model of four-high rolling mill stand

The limit state function of the composite tensile and bending strength of column is listed as follows:

$$G(t,\omega)=[\sigma_P]-\frac{500}{x_1x_2}+\frac{750(x_1+0.59)}{x_1^2x_2}\times\left[\frac{1}{1+\dfrac{(x_3+x_5+4.3)x_4x_3^3x_6x_5^3}{(x_1+0.59)x_2x_1^3(x_4x_3^3+x_6x_5^3)}}\right]$$

where the data of σ_p can be obtained by MATLAB normal sample method during 100 months, which is shown in Fig. 12. Considering the degradations of x_4, x_5, x_6 over time, leads to

$$x_4=x_4\pm\varepsilon\,,\; x_5=x_5e^{-0.001*t}\,,\; x_6=x_6e^{-0.001*t}$$

During MRDO optimization process, set ΔX=0.01X, x_4, x_5, x_6 are normal distributed in the module of time-varying reliability sensitivity and corresponding standard deviations are 0.015,0.03,0.014 (units: m).

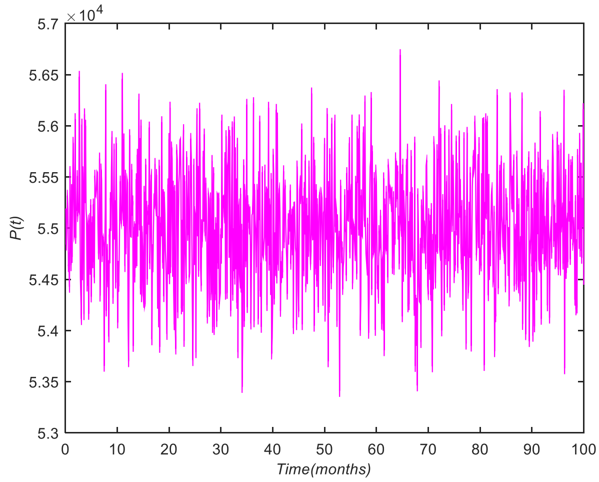


Fig. 12 The data of σ_p

The optimization results of conventional MDO method and the proposed method are listed in Table 2. The corresponding iteration processes of objective function and constraints are shown in Fig.13 and Fig. 14, respectively. The time-varying sensitivity indexes of these two methods are shown in Fig. 15. The sensitivity proportions of x_4, x_5, x_6 are shown in Fig. 16.

Tab.2 Optimization results （unit:mm）

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	f

Original design	303	260	817	327	789	318	420	0.91
Novel design	265	263	660	267	1000	404	416	0.94

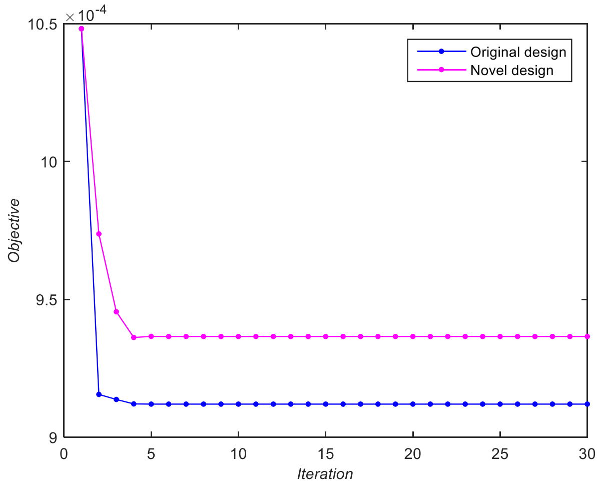


Fig. 13 The iteration process of objective function

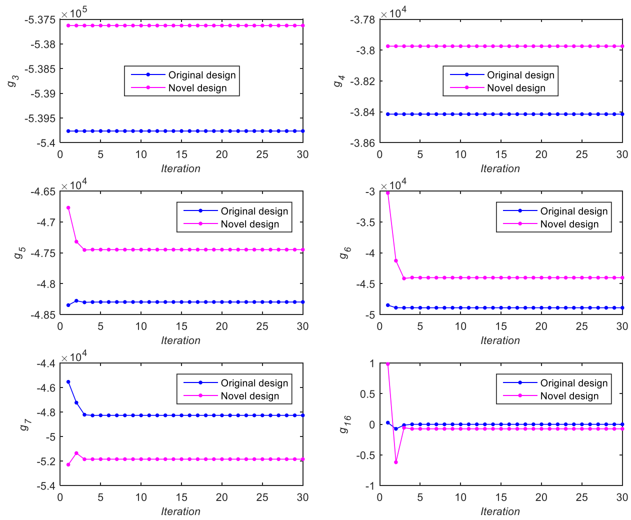


Fig. 14 The iteration process of constrains

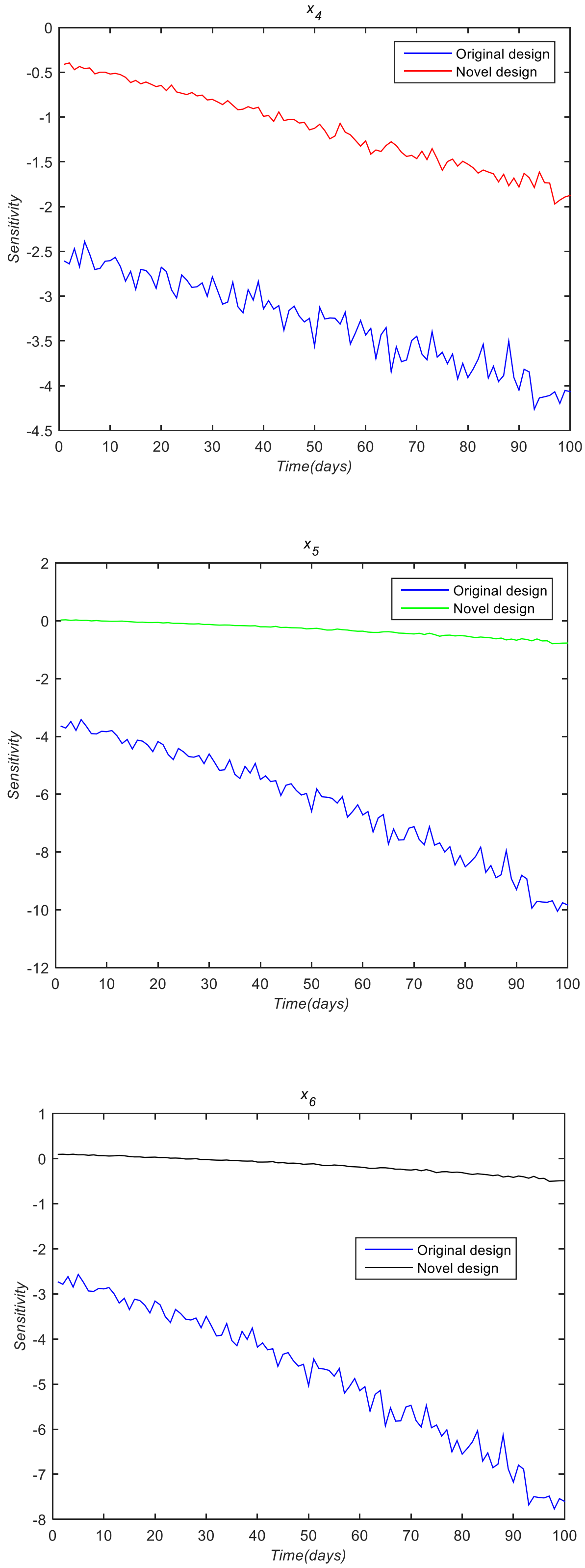


Fig. 15 The comparison of time-varying sensitivity indexes

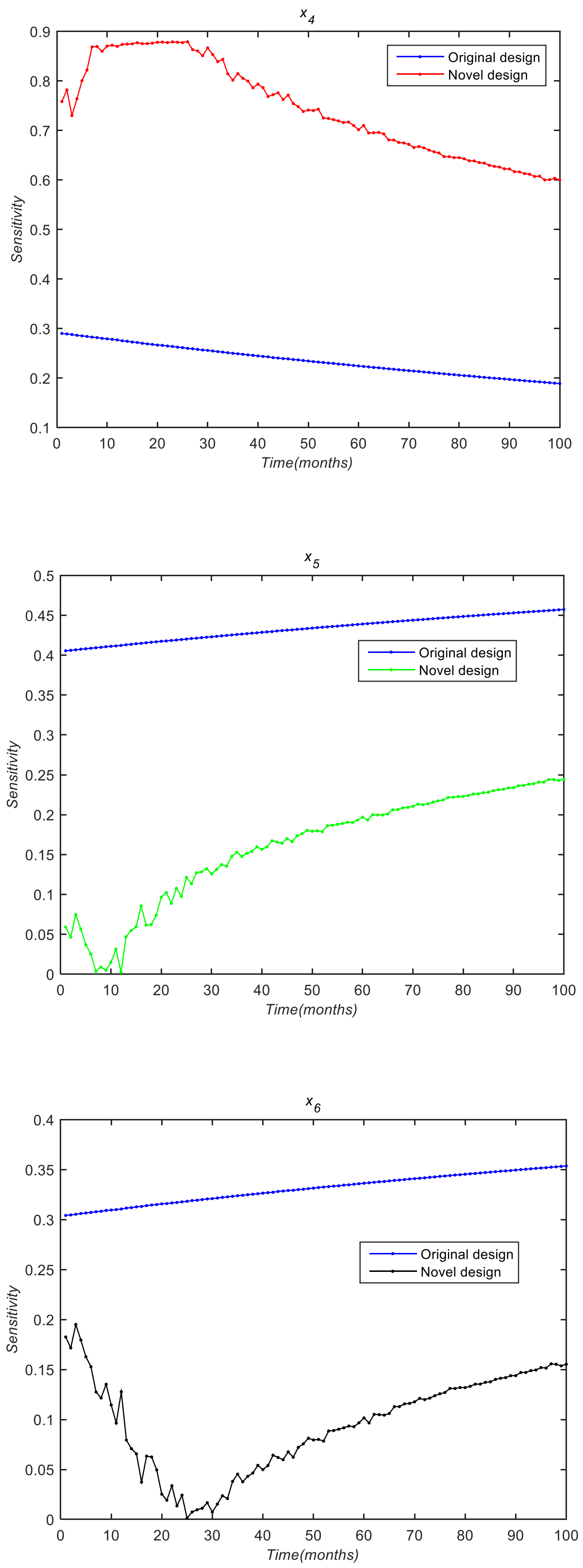


Fig.16 Comparison of sensitivity proportions

From Tab.2, the optimization results of the proposed method increases by 3.3% than that of conventional design method. However, note from the Fig. 15 that the time-varying sensitivity index of conventional method increases over time, which means time-varying uncertainties x_4, x_5, x_6 have shown great influences on performance function. On the other hand, although the time-varying sensitivity indexes of the proposed method slowly increases over time, the values are rather smaller and has shown smaller fluctuation, which means time-varying uncertainties x_4, x_5, x_6 has small influence on performance function. it's worth noting from Fig.16 that the sensitivity proportions of time-varying uncertainties x_5, x_6 obtained from conventional MDO method are getting bigger over time, the sensitivity proportion of time-varying uncertainties x_4 is getting smaller over time, which means time-varying uncertainties have more effects on performance function over time. The sensitivity proportions of x_5, x_6 obtained by the proposed method are smaller than that of conventional method. Sensitivity proportion trends to time-insensitive x_4, which means our optimization result is robust. From this engineering example, it's worth noting that time-varying uncertainties have shown a great influence on the performance of the system. In practical engineering optimization, thus, the effects of time-varying uncertainties will be properly considered using the proposed model in this paper.

5. Conclusions

In this paper, the time-varying uncertainties of complex mechanical systems are investigated, a new method for time-varying reliability sensitivity analysis is proposed; In addition, the propagations of time-varying uncertainties in MDO are analyzed, a MRDO model is established by using IUP method; the framework of MRDO under time-varying uncertainties is established, then mathematical and engineering examples are introduced to verify the accuracy and effectiveness of the proposed method. Due to the complexity of time-varying uncertainties, the proposed method is conservative to some extent; the accuracy and efficiency of the proposed method still need to be improved.

Acknowledgments

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Huanwei Xu is currently an associate professor at the School of Mechatronics Engineering of the University of Electronic Science and Technology of China in Chengdu, Sichuan,China. He received a PhD degree in Mechanical Engineering from Dalian University of Technology in Dalian, China. He has published 20 journal papers, and his research interests include multidisciplinary design optimization, robust design and reliability design.

Mufeng Li is studying at the University of Electronic Science and Technology of China as a graduate student. His main research interests include in the area of robust design.

Wei Li is currently studying at the School of Mechatronics Engineering of the University of Electronic Science and Technology of China as a graduate student. His main research interests include uncertainty quantification and propagation, sensitivity analysis, MDO, etc.

Cong Hu is currently studying at University of Electronic Science and Technology of China as a graduate student, under the guidance of professor Huanwei Xu.His main research interests include time-dependent based multidisciplinary optimization design.

Suichuan Zhang is studying at University of Electronic Science and Technology of China as a graduate student. His mainly reseach direction is structure strength optimization, life prediction and multidisciplinarydesign optimization.

Xin Wang is currently at University of Electronic Science and Technology of China as a graduate student. His main research interest includes in the area of engineering design and optimization.